Question 1

What determines the choice of the contour in the complex integral calculation of page 291-292?

The contour has to be chosen so that the integral vanishes along the contour. Because of the choice of the Fourier integral below Eq. (13.31), and the fact that $r$ is always positive, the contour in the upper half-plane is the one along which the integral vanishes, whether considering $G_+$ or $G_-$. 

Question 2

What does the Born approximation consist of and when do you expect it to be valid?

The Born approximation is an iteration procedure where the potential energy term is treated as a perturbative source, whose effect is first neglected to obtain an approximate solution, and then included iteratively by feeding the approximate solution as the source term. One expects the Born approximation to be valid when the incoming energy (in plane waves) is large corresponding to the interaction energy (potential).

Question 3

Why is it not inconsistent to keep different orders of $r'/r$ in the denominator and exponent of Eq. (13.39) when taking the asymptotic limit?

Whereas an expansion in the denominator leads to additional terms (the smaller ones vanishing asymptotically), an expansion in an exponent leads to a product of factors. Consistency should be decided at the end of the calculation of the expression, not in the argument.

Question 4

Does the Born approximation make sense for non spherically symmetric potentials? If not, why not? And if so, does anything change?

Yes, it does make sense. However the integral is typically more involved. The scattering amplitude now depends on more than the magnitude of the momentum transfer.

Question 5

What very remarkable fact exist for the cross section of the Coulomb potential?

The classical calculation, the quantum mechanical calculation, and the Born approximation all give exactly the same differential cross section.