Physics 321 - Take-home Test #1

Problem 1
In HW#2, we came up with four equations that could simultaneously be solved to find the motion of a relativistic object. We will use these equations to solve for the motion of an electron in an electrostatic accelerator.

(A - 10 points) Simultaneous Equations in Mathematica
The equation for a simple harmonic oscillator is \( m \frac{dv}{dt} = -kx \). Note that this is equivalent to two simultaneous equations: \( m \frac{dv}{dt} = -kx \), \( \frac{dx}{dt} = v \). Let us assume that at \( t=0 \), \( x=0 \) and \( v=v_0 \).
Let \( m=0.300 \) kg, \( v_0=1.64 \) m/s, and \( k=4.75 \) N/m. Find the maximum displacement of the system in meters.

(B - 40 points) An electron is accelerated from rest through a uniform potential increase of 2.17 MV over a distance 1.60 m. What is the final velocity and kinetic energy of the electron? Evaluate the kinetic energy in Joules and MeV. 1 MeV = 1.602e-13 J
The electric field is then \( 2.17e6 \) V/1.60 m, and the force is the electron charge times this. Use the relativistic equations from HW 2 to solve for these values.

Problem 2
(A) If a baseball were hit in a vacuum, at what angle should it be hit to maximize its range? If the maximum range is 92.0 m, at what velocity does it leave the bat?
Consider the range to be the distance traveled when the ball returns to the height at which it was hit. That is, you do not need to consider the height of the batter.

(B) Add linear and quadratic drag terms and solve the system of equations numerically. Use the value of \( v_0 \) found in part (A) for your initial speed. Plot \( y(t) \) as a function of \( x(t) \). By trial and error, find the angle to the nearest degree that gives the greatest range. Use ParametricPlot and NDSolve over appropriate ranges of time. You may restrict these ranges as you wish to zero in the angle that gives the largest range.

(C) Now repeat part (B) assuming that a steady wind of 30 mph is blowing in the -x direction. That it, it is blowing the ball directly back toward the batter.

Information:
A baseball has a diameter of 7.40 cm and a mass of 145 g. Treat it as a smooth sphere. Use linear and quadratic drag coefficients from section 2.1 of the text.

(A - 40 points) You should solve equations of motion in both the x and y directions.
(B - 40 points) Write the x (horizontal) and y (vertical) components of the linear and quadratic drag terms in terms of \( vx(t) \) and \( vy(t) \), the components of the velocity. Solve for the equations of motion using the "\( m \frac{dv}{dt}=F \), \( \frac{dx}{dt}=v \)" form of Newton's Second Law.
(C - 20 points) Write the $x$ (horizontal) and $y$ (vertical) components of the linear and quadratic drag terms in terms of $v_x(t)$ and $v_y(t)$, the components of the velocity. Solve for the equations of motion using the "$m \frac{dv}{dt} = F, \frac{dx}{dt} = v$" form of Newton's Second Law.