Physics 321 – Final Review

Motion of Solids
a. Find inertia tensor
b. Know how to diagonalize it (you won’t need to actually do it). Interpret the eigenfunctions and eigenvalues. 
\[ I\ddot{\omega} = I\ddot{\omega}, \quad \det(I - I) = 0. \]
c. Euler’s equation, \( \ddot{r} = \ddot{L} + \ddot{\omega} \times \dot{L} \). All vectors are in the body basis. Simplify for zero torque and \( I_{11} = I_{22} \).
d. Euler’s angles, \( \theta, \phi, \psi \). Know the meaning of each. Find the orientation of a basis set rotated by simple angles such as 90º. Describe \( \theta, \phi, \psi \)
e. Football. Know the constants of motion (\( \ddot{L}, \ddot{t}, \ddot{L}, \theta, \alpha \)). Which angles are ignorable?
f. Know how torque makes the motion of a top different from that of a football. – Nutation.

II. Coupled Oscillators
a. Find the Lagrangian and equations of motion.
b. Know the definition of normal modes.
c. Normal mode analysis. Change the equations of motion into a matrix equation. Know there is one mode per degree of freedom. Describe the normal modes if you know the eigenvalues and eigenvectors. Be able to find eigenvalues and eigenvectors for a simple 2 \times 2 system.
\[ -\omega^2 \mathbf{M} = -K \ddot{\mathbf{x}}, \quad M^* K \ddot{x} = \omega^2 x. \]
If \( \mathbf{M} = \begin{bmatrix} A & B \\ C & D \end{bmatrix}, \mathbf{M}^{-1} = \frac{1}{AD - BC} \begin{bmatrix} D & -B \\ -C & A \end{bmatrix}. \]
d. Know that the general solution is a linear combination of the normal modes.

III. Scattering
a. Know the definitions of \( \sigma, \frac{d\sigma}{d\Omega}, \Delta \Omega = \frac{\Delta A}{r} = \sin \theta \Delta \theta \Delta \phi \).
b. Know the definitions of the impact parameter and scattering angle.
c. Be able to do conservation of energy and momentum and be able to transform from lab to cm frames.
d. Given \( b(\theta) \), be able to find \( \frac{d\sigma}{d\Omega} : \frac{d\sigma}{d\Omega} = b \left| \frac{db}{d\theta} \right| \)

IV. Chaos
a. Nonlinear equations often lead to harmonics and frequency dependent amplitudes.
b. Know how to use a phase space (state space) plot. Recognize that frequency doubling can be a characteristic of the onset of chaos.
c. Know that chaos is characterized by non-repeating cycles in phase space orbits and great sensitivity of initial conditions.