Lesson 9 – Dipoles and Magnets

9.0 Introduction

In this chapter we will learn about an assortment of topics that at first glance seem dissimilar, but which are actually quite closely related. In the first part, we will learn about the force on a current-carrying wire, electric and magnetic dipoles, and how dipoles behave in uniform and nonuniform fields. In the second half, we will talk about different ways in which matter can respond to external magnetic fields. We'll learn about paramagnetism, diamagnetism, and ferromagnetism and discuss some properties of permanent magnets.

9.1 The Force on a Current-Carrying Wire

We know from the Lorentz Force Law that a charge moving in a magnetic field experiences a force

$$\vec{F} = q\vec{v} \times \vec{B}.$$

In a current-carrying wire, the positive ions are stationary (relative to an observer at rest with respect the wire), but the electrons are moving. This means that the electrons in the wire experience a magnetic force, although the positive ions do not. To see how this affects a wire, let us think of the arrangement shown below.
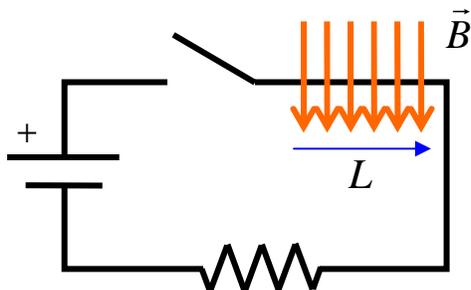

Figure 9.1 A section of wire in a magnetic field.

When the switch is closed, charges (think of positive charges) begin moving from the positive terminal of the battery. As the charges pass through the region where the magnetic field is located, they experience a force into the screen. The charges move freely in the wire, pushed along to the right by the battery and to the back of the wire by the magnetic field. However, the charges can only move a bit toward the back of the wire before they are repelled by other charges collecting near the back side of the wire. At this point, the force exerted on the charges is transferred to the ions in the wire. That is, the entire wire feels a force equal to the sum of the forces acting upon each charge on the wire.

Think About It

Would the force on the wire be any different if we took into consideration the fact that the current is actually caused by electrons moving to the left through the region of the magnetic field?

Now let's do a little algebra. Let λ be the number of charge carriers (valence electrons) per unit length in the wire, L be the length of the region where the magnetic field is located, and N be the total number of electrons in this region. Then $N = \lambda L$ and the total force on the wire is

$$F = NevB = \lambda LevB$$

where v is the drift velocity of electrons in the wire. Now, let T be the time it takes the electrons to go a distance L in the wire, so $v = L / T$.

The current in the wire is then

$$i = \frac{Ne}{T} = \frac{\lambda Le}{T} = \lambda ev \ .$$

Putting this all together

$$F = \lambda LevB = iLB$$

We can generalize this to the formula to something that is equivalent to the Lorentz Force Law but for currents rather than individual particles:

(9.1) $\vec{F} = i\vec{L} \times \vec{B}$

where

$\quad \vec{F}$ is the force on a segment of wire in newtons (N) .
$\quad \vec{L}$ is the length of the wire segment in meters (m).
$\quad\quad$ The direction of $\vec{L}$ is the direction of the current.
$\quad i$ is the current in the wire in amperes (A).

A little bad calculus can be used as a mnemonic device to help remember this relationship:

$$d\vec{F} = dq\,\vec{v} \times \vec{B} = dq\frac{d\vec{L}}{dt} \times \vec{B} = \frac{dq}{dt}d\vec{L} \times \vec{B} = id\vec{L} \times \vec{B}$$

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Here $d\vec{L}$ is the bit of force on a small charge dq, and $d\vec{L}$ is a little segment of wire pointing in the direction of the current.

Things to remember:
• The force on a segment of current-carrying wire is $\vec{F} = i\vec{L} \times \vec{B}$

9.2 Magnetic Dipoles

Many times an entire circuit is located within a magnetic field. To see how the field affects a circuit, let us take a particularly simple case of a current traveling around a square loop within a uniform magnetic field. The length of each side of the loop is taken to be a.
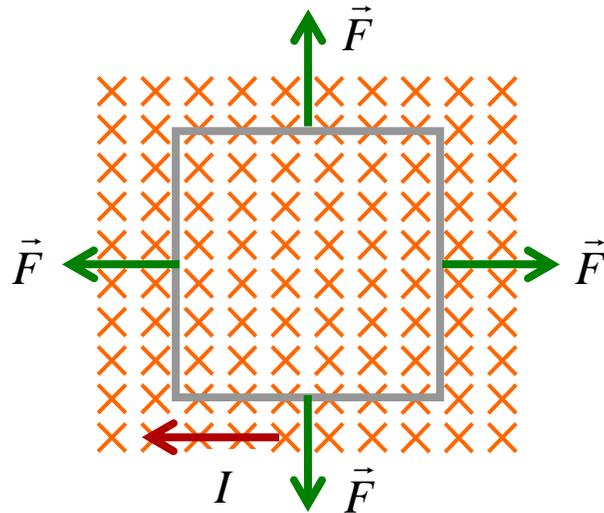
Figure 9.2 A current-carrying loop of wire in a uniform magnetic field.

Such a Such a loop, with a magnetic field passing perpendicularly through the loop, is shown in Fig. 9.2. By applying Eq. (9.1) to each section of the loop, we find that the force in each segment of the wire is directed away from the center of the loop. The total force on the loop is zero. Furthermore, there is no torque tending to rotate the loop.
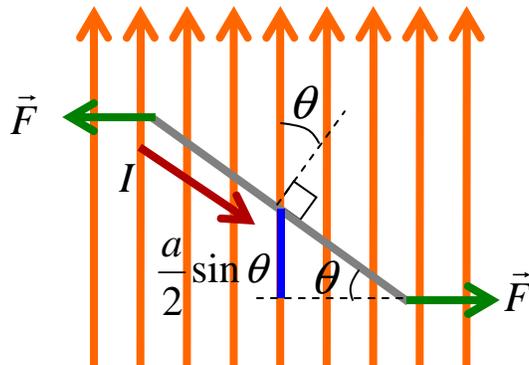
Figure 9.3 A current-carrying loop tilted at an angle θ to the magnetic field.

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Now let's look at a similar loop, but with the magnetic field passing through the loop at an angle. We call the angle between the magnetic field and the *normal* to the loop, θ. A view of the loop seen edge-on is shown in Fig. 9.3. In this case, the forces on the sides of the loop point again away from the center of the loop. (These forces are not shown in the figure.) Hence, they cancel out as before. The forces on the top and bottom segments of the wire are still in opposite directions, as shown, but they are no longer directed away from the center of the loop. The net result is that the total force on the loop is still zero, but now there is a net torque on the loop, tending to rotate it in a counterclockwise direction. The magnitude of the torque from each force is the product of the moment arm, $a/2 \sin \theta$, and the force F. The total torque is just twice this amount:

$$\tau = 2\frac{a}{2}\sin\theta \; F = a\sin\theta \; iaB = ia^2 B\sin\theta .$$

We now define a quantity called the "magnetic dipole moment" of the current loop. This is a vector represented by the Greek letter mu, $\vec{\mu}$. The magnitude of the magnetic dipole moment is just the product of the current in the loop and the area of the loop. The direction is taken to be the normal to the loop; however, this is still ambiguous as there are two different possibilities of directions for the normal. For example, in Fig. 9.2, the normal could be either into the screen or out of the screen. In order to resolve this ambiguity, we establish a rule.

Right-hand Rule for the
Magnetic Dipole Moment

Curl the fingers of your right hand in the direction current flows around the loop. The direction of the magnetic dipole moment is in the direction your thumb points.
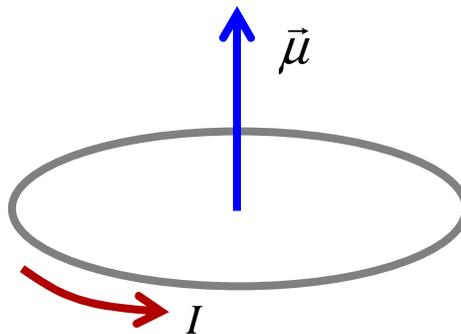

Figure 9.4 The right-hand rule for the magnetic dipole

We can write the magnetic moment as:

Magnetic Dipole Moment

(9.2) $$\vec{\mu} = i\vec{A}$$

where

$\vec{\mu}$ is the magnetic dipole moment in ampere-meters² (Am^2).

i is the current in amperes (A).

$\vec{A}$ is the directed are in square meters (m^2).

The direction of $\vec{A}$ is given by the right hand rule above.

In terms of the magnetic dipole moment, we see that we can easily write the torque on a current loop as:

Torque on a Magnetic Dipole

(9.3) $$\vec{\tau} = \vec{\mu} \times \vec{B}$$

where

$\vec{\tau}$ is the torque in newton-meters (Nm).

$\vec{\mu}$ is the magnetic dipole moment in ampere-meters² (Am^2).

$\vec{B}$ is the magnetic field in tesla (T).

This equation summarizes three important facts: 1) The torque on a dipole in a uniform field is proportional to the magnitude of the dipole moment and to the magnitude of the magnetic field. 2) The torque on a dipole in a uniform field is greatest when the dipole is perpendicular to the field and zero when the dipole is parallel or antiparallel to the field. 3) The torque on the dipole tends to rotate the dipole moment into the direction of the magnetic field.

If a dipole is oriented so that its dipole moment is in the direction of the field, it experiences no torque. To rotate the dipole by an angle θ, a torque is required and work must be done. In analogy with work where $W = \int F dx$, we have

$$W = \int_{\theta_1}^{\theta_2} \tau \, d\theta = \int_{\theta_1}^{\theta_2} \mu B \sin\theta \, d\theta = -\mu B (\cos\theta_2 - \cos\theta_1)$$

If we rotate the dipole at a constant speed, the work done to the dipole results only in a change in potential energy. We can then define the potential energy of a magnetic dipole in a uniform magnetic field by:

$$U = -\mu B \cos\theta + C$$

where C is a constant of integration. For convenience, we can let the constant be equal to zero. Then we can write this in the simple form:

Energy of a Magnetic Dipole

(9.4)
$$U = -\vec{\mu} \cdot \vec{B}$$

where

  U is the potential energy of a magnetic dipole in joules (J).
  $\vec{\mu}$ is the magnetic dipole moment in ampere-meters^2 (Am^2).
  $\vec{B}$ is the magnetic field in tesla (T).

Note that the potential energy is zero when the dipole is *perpendicular* to the field. This, however, does **not** mean that the potential energy is a minimum when dipole moment and the field are perpendicular! The minimum value of the potential energy is $U = -\mu B$ when the dipole is parallel to the field, at $\theta = 0$. The maximum value of the potential energy is $U = +\mu B$ when the dipole is antiparallel to the field, at $\theta = \pi$. No go back and reread this paragraph – many students get confused by this!
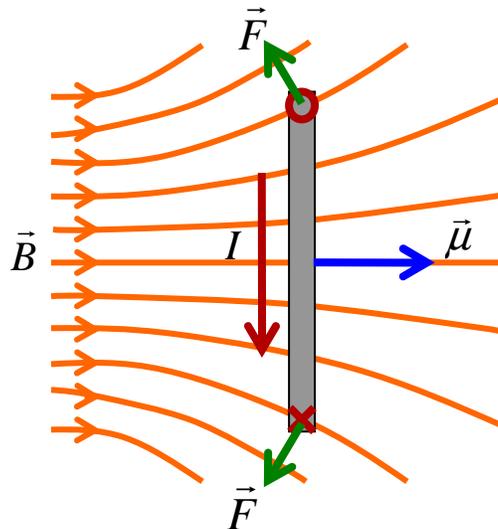

Figure 9.5 A current-carrying loop of wire in a nonuniform magnetic field.
$\vec{\mu}$ is parallel to $\vec{B}$ at the center of the loop.

So far, we have looked at what happens to magnetic dipoles when they are placed in uniform magnetic fields. What happens when the field is not uniform? Consider a circular current loop shown edge-on in Fig. 9.5. Current goes into the screen at the bottom of the loop and comes out of the screen at the top of the loop. The direction of the magnetic field varies somewhat across the loop; however, at the center of the loop the dipole moment and the magnetic field are parallel. The forces on the loop at the top and bottom are shown in the figure. If we add the forces from all the sections of the loop, we see that the net force must be to the left. In general, it can be shown that when the dipole moment and the field are parallel, the dipole

experiences a force in the direction of increasing magnetic field. On the other hand, if the dipole moment is antiparallel to the field, the net force is toward smaller fields.

However, if the dipole moment is neither parallel nor antiparallel to the magnetic field, the dipole also experiences a torque that tends to align the dipole moment with the field. In general, the motion can be very complicated, but usually, there are frictional forces that damp the oscillatory motion that can ensue, and the dipole eventually aligns with the external field and is attracted into the region where the field is stronger.
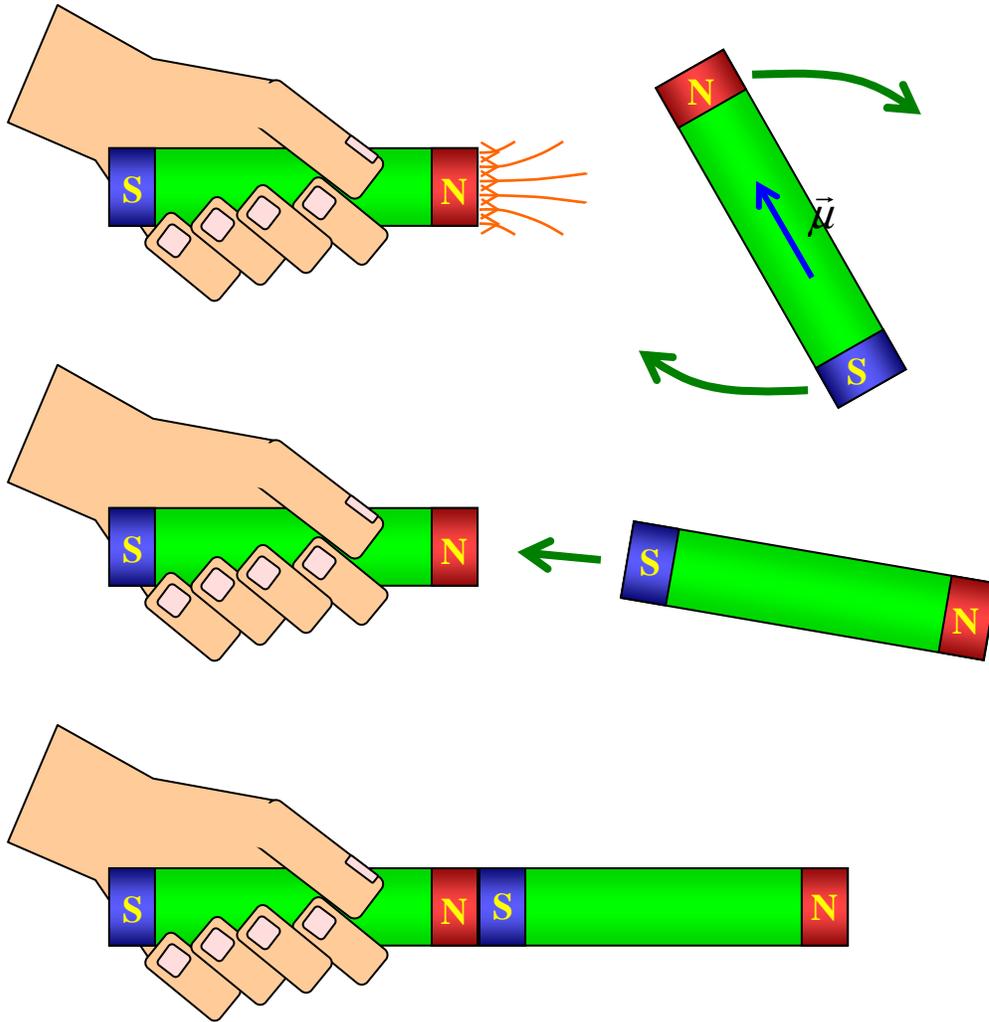
Figure 9.6 The interaction of two permanent magnets.

At this point, you are probably asking yourself why we bother learning about what happens to current loops in magnetic fields. The best answer is that nature is full of tiny current loops: electrons orbiting nuclei in atoms. Permanent magnets are a collection of many dipoles. The forces between permanent magnets can be quite complicated because there are so many dipoles interacting; however, in principle these forces can be understood in terms of dipoles. The dipole moment of a permanent magnet goes from the south pole to the north pole (inside the magnet), as shown in Fig. 9.6. When we place the north pole of one magnet near another magnet,

the dipole moment of the second magnet aligns with the field of the first magnet, so the south pole of the second magnet faces the north pole of the first magnet. The second magnet then feels a force in the direction of increasing magnetic field, toward the first magnet.

Things to remember:
• The magnetic dipole moment is given by $\vec{\mu} = i\vec{A}$ where the direction of A is given by the right-hand rule: fingers are I, thumb is A.
• The torque on a magnetic dipole in a magnetic field is $\vec{\tau} = \vec{\mu} \times \vec{B}$.
• The potential energy of a magnetic dipole in a magnetic field is $U = -\vec{\mu} \cdot \vec{B}$.

9.3 Electric Dipoles

We have already encountered electric dipoles in Lesson 6 when we learned how dipoles in dielectric materials can align to decrease the electric field and thereby increase the capacitance of a capacitor. The simplest form of an electric dipole is a charge +q joined by a rigid rod to a charge –q. We will call the distance between the charges ℓ.

If an electric dipole is placed in a uniform electric field, as shown in Fig.9.7, the two charges feel equal forces in opposite directions. The net force on the dipole is zero. However, there is a net torque on the dipole. The torque on each charge is given by the product of the force, $F = qE$, and the moment arm, $\ell \sin\theta / 2$, where θ is the angle between the rod and the field, as shown in the figure. The total torque on the dipole is

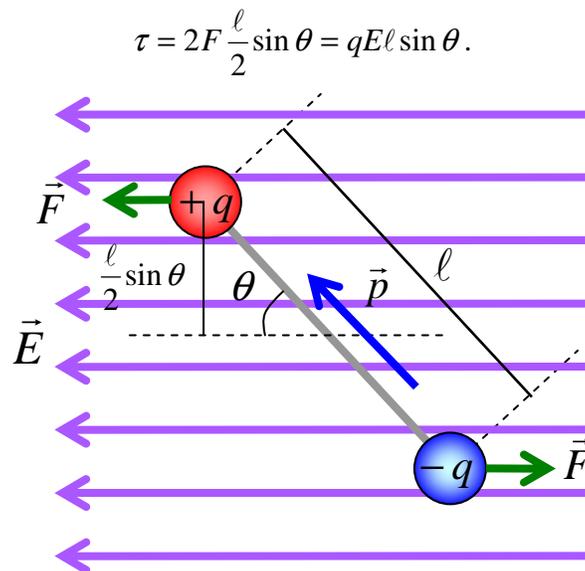$$\tau = 2F\frac{\ell}{2}\sin\theta = qE\ell\sin\theta.$$

Figure 9.7 An electric dipole in a uniform electric field.

We define an "electric dipole moment" $\vec{p}$ as vector of magnitude $p = q\ell$ pointing from the negative charge to the positive charge, as shown in Fig. 9.7. Thus:

> ## Electric Dipole Moment
>
> $$\vec{p} = q\vec{\ell}$$
>
> where
> > $\vec{p}$ is the electric dipole moment in units of *Cm*,
> > q is the magnitude of the charge on each end of the dipole, in *C*
> > $\vec{\ell}$ is the vector distance from the negative charge to the positive charge, in *m*

In terms of $\vec{p}$, the torque on the dipole is:

> ## Torque on an Electric Dipole
>
> (9.5) $\vec{\tau} = \vec{p} \times \vec{E}$.

Note that this equation is very similar to Eq. (9.3). Also note that the direction of $\vec{p}$ is chosen so that the torque tends to rotate $\vec{p}$ into $\vec{E}$.

   Just as in the case of magnetic dipoles, the potential energy of an electric dipole is given by the work required to rotate it in the electric field.

$$W = \int_{\theta_1}^{\theta_2} \tau \, d\theta = \int_{\theta_1}^{\theta_2} pE \sin\theta \, d\theta = -pE(\cos\theta_2 - \cos\theta_1)$$

This leads to the relationship

> ## Potential Energy of an Electric Dipole
>
> (9.6) $U = -\vec{p} \cdot \vec{E}$

which resembles Eq. (9.4) for magnetic dipoles.

   The behavior of electric dipoles in non-uniform fields also has similarities to the behavior of magnetic dipoles in non-uniform fields. The dipole experiences a torque that tends to align the dipole moment with the electric field, and when aligned, the dipole experiences a net force in the direction the field is increasing.
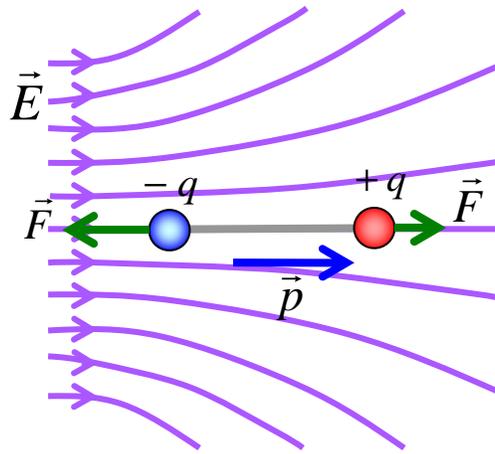
Figure 9.8 An electric dipole in a non-uniform electric field.

Things to remember:

• The electric dipole moment is defined by $\vec{p} = q\vec{\ell}$ where the direction of ℓ is from $-q$ to $+q$.

• The torque on a electric dipole in a magnetic field is $\vec{\tau} = \vec{p} \times \vec{E}$.

• The potential energy of a magnetic dipole in a magnetic field is $U = -\vec{p} \cdot \vec{E}$.

9.4 Atomic Dipoles, Domain Alignment, and Thermal Disalignment

In the early 20th Century, scientists discovered that individual electrons also behave like little magnetic dipoles. They initially thought of the electron as a very tiny sphere of charge spinning like the earth about an axis. Hence, electrons were said to have "spin." However, there were inconsistencies between the spinning sphere model and experimental observations. In particular, as far as we can now tell, the electron is a point particle – a particle with no size at all. We now think of spin as an intrinsic characteristic of an electron, one that obeys the rules of angular momentum. But we really don't know what spin is and we really don't know why electrons behave like little magnets. We do know that many other particles also have magnetic dipole moments. Even neutrons, which are neutral, behave like little magnets. In fact, the magnetic dipole moment of a neutron suggests that it has positive charge toward the center and negative charge on the outside.

Since normal matter is made up of protons, neutrons, and electrons, each of which are little magnets themselves, it is not surprising that atoms are little magnets as well. But in addition to the magnetic dipole moment that arises from particle spin, there is also a contribution to the total dipole moment of an atom from the orbital angular momentum of electrons. That is, each electron in an atom produces a tiny current loop that adds to the overall magnetic dipole moment of the atom. As it turns out, protons, neutrons, and electrons are all in orbitals occupied, where possible, by two particles with opposite spins. The magnetic dipole moments of these paired particles then cancel. Furthermore, the orbital angular momentum of electrons in different orbitals also tends to cancel. This generally leaves atoms with very small, but non-zero, magnetic dipole moments.

When solids are arranged in crystals, there are two competing effects that usually dominate a material's response to magnetic fields. These are domain alignment and thermal disalignment.

To understand domain alignment, we can think of atoms as little magnets. If we take a stack of magnets, north poles attract south poles, and the magnets tend to align as shown in Fig, 9.9. The dipole moments of atoms in a crystal prefer to be aligned with all their dipole moments pointing in the same direction. In materials where this effect predominates, large numbers of atoms are aligned in "domains" where the dipole moments are all parallel. Such material are called "ferromagnets." Often there are many small domains in a crystal with the direction of the dipole moment in different domains randomly oriented, as illustrated in Fig. 9.10. An example of such a material would be an iron nail. If the domains, however, have a predominant direction as shown in Fig. 9.11, the material is a permanent magnet.
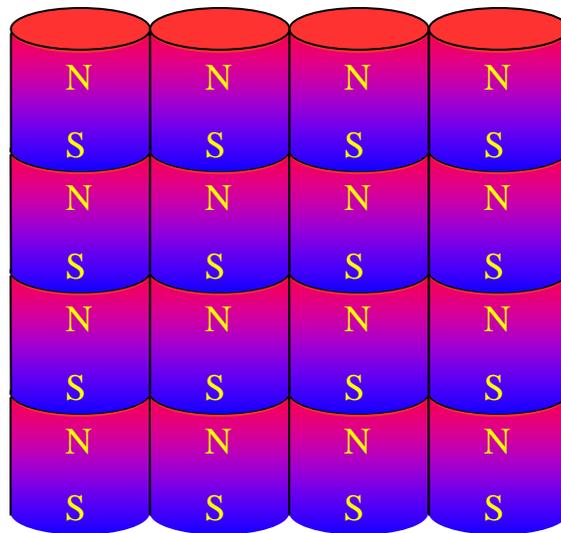

Figure 9.9 Magnets tend to align with their dipole moments in the same direction.

On the other hand, atoms in crystals have vibrational energy. As atoms vibrate, the orientation of their magnetic dipole moments tends to be randomized. This is called "thermal disalignment." In almost all materials, magnetic domains can not be established because thermal disalignment overshadows domain alignment. Even ferromagnetic materials cease to be ferromagnetic when heated enough. The temperature at which a given ferromagnetic substance ceases to have aligned domains is call the "Curie Temperature" of the material.
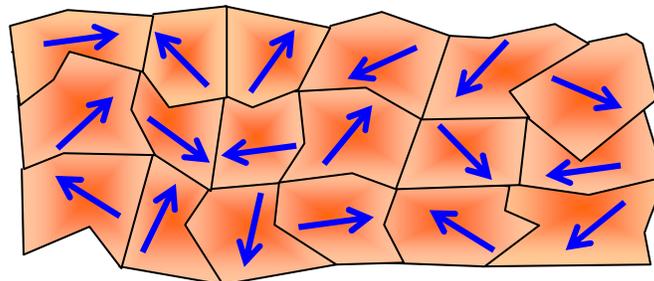
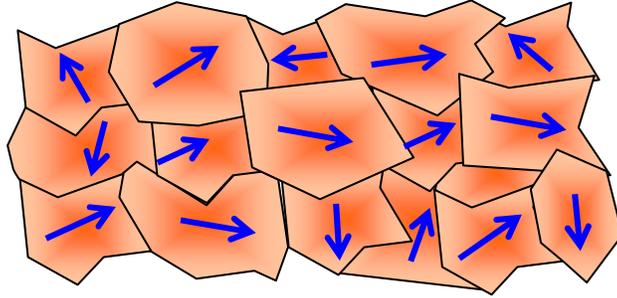Figure 9.10 Magnetic domains in non-magnetized soft iron.

Figure 9.11 Magnetic domains in a permanent magnet

Things to remember:
• Elementary particles have magnetic dipole moments associated with their spin.
• Atoms have dipole moments that arise from spin and the orbital motion of electrons.
• Atomic dipoles tend to align with one another.
• Thermal agitation tends to disalign the dipoles.

9.5 Matter in External Magnetic Fields

When matter is placed in an external magnetic field, its behavior falls into one of three categories: ferromagnetic, paramagnetic, or diamagnetic. We have already seen that ferromagnetic materials are those in which domains can remain aligned in spite of thermal effects. We will consider characteristics of ferromagnetic materials in more detail in the next section.

When a small permanent magnet is placed in a uniform magnetic field, it feels a torque that tends to align its magnetic moment with the external field. Once the magnetic dipole moment points in the direction of the external field, rotational inertia will carry the magnet past the position of minimum energy. Somewhat like a pendulum or a mass attached to a spring, the dipole will continue to oscillate until frictional forces damp out the oscillatory motion. At that point, the dipole will remain aligned in the field, as illustrated in Fig. 9.12.
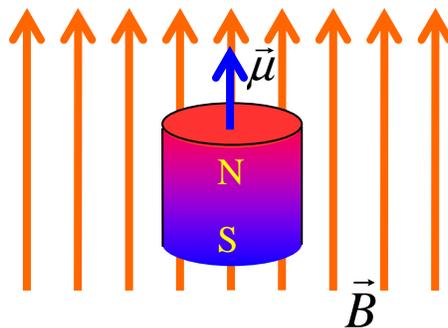

Figure 9.12 A permanent magnet or a paramagnetic atom
in an external magnetic field.

Most materials have atoms that do precisely this, except that thermal effects regularly knock the direction of the dipole around. Even in spite of thermal effects, the atoms do align

preferentially with their dipole moments in the direction of the external field. Such materials are called "paramagnetic."

We can characterize just how much the atoms or molecules of a material tend to align with the external field by a quantity called the "magnetic susceptibility" of the material. Before we define magnetic susceptibility, however, we need to clearly understand the origin of what we call the "internal" magnetic field. All the atoms in a material contribute to the total magnetic dipole moment of a sample. The magnetic dipole moment per unit volume is a quantity called the "magnetization" of a material. We may write this as:

$$\vec{M} = \frac{\sum_{i=1}^{N} \mu_i}{Volume}$$

The internal magnetic field of a sample is related to the magnetization by the relationship

(9.7) $$\vec{B}_{int} = \mu_0 \vec{M} \; .$$

The magnetic susceptibility relates the strength of the internal field to the external field. That is,

(9.8 Magnetic susceptibility) $$\vec{B}_{int} = \chi \vec{B}_{ext}$$

where
χ (chi) is the magnetic susceptibility. It is dimensionless.

The total magnetic field inside of a material is the sum of the internal and external magnetic fields.

The magnetic susceptibilities of paramagnetic materials range from about $+10^{-5}$ to $+10^{-3}$. As you can see by these numbers, paramagnetism is a typically a very weak effect. Because paramagnetic materials are weak magnetic dipoles, they are weakly attracted from regions of smaller magnetic field into regions of larger magnetic field. (See Fig. 9.6.)

It turns out that other materials behave in exactly the opposite way in external magnetic fields. That is, the magnetic dipole moments prefer to align in the direction opposite to the magnetic field, as shown in Fig. 9.13. Such materials are called "diamagnetic." This behavior is adequately explained only by treating atoms quantum mechanically.
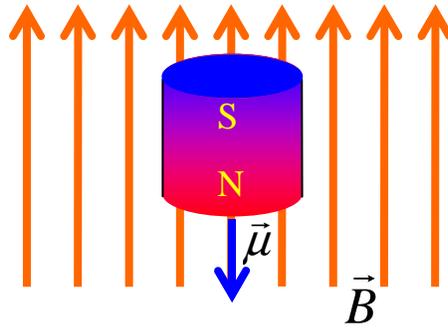
Figure 9.13 A diamagnetic atom in an external magnetic field.

Diamagnetic materials have susceptibilities that are negative and very small in size, ranging generally from about -10^{-6} to -10^{-4}. Because the diamagnetic "magnets" are aligned opposite to the field, they are repelled weakly from regions of larger magnetic fields into regions of smaller magnetic field.

Things to remember:
• Magnetic susceptibility is defined by $\vec{B}_{int} = \chi \vec{B}_{ext}$.
• See the table at the end of the next section.

9.6 Ferromagnetism

Ferromagnetic materials have by far the most interesting behavior in external magnetic fields. Even in moderate external fields, the domain boundaries shift to allow domains aligned with the external field to increase in size while domains aligned in other directions shrink in size. Because of this, the internal field can become much larger than the external field. For ferromagnetic materials, magnetic susceptibilities are roughly 10^{+3} to 10^{+5}.
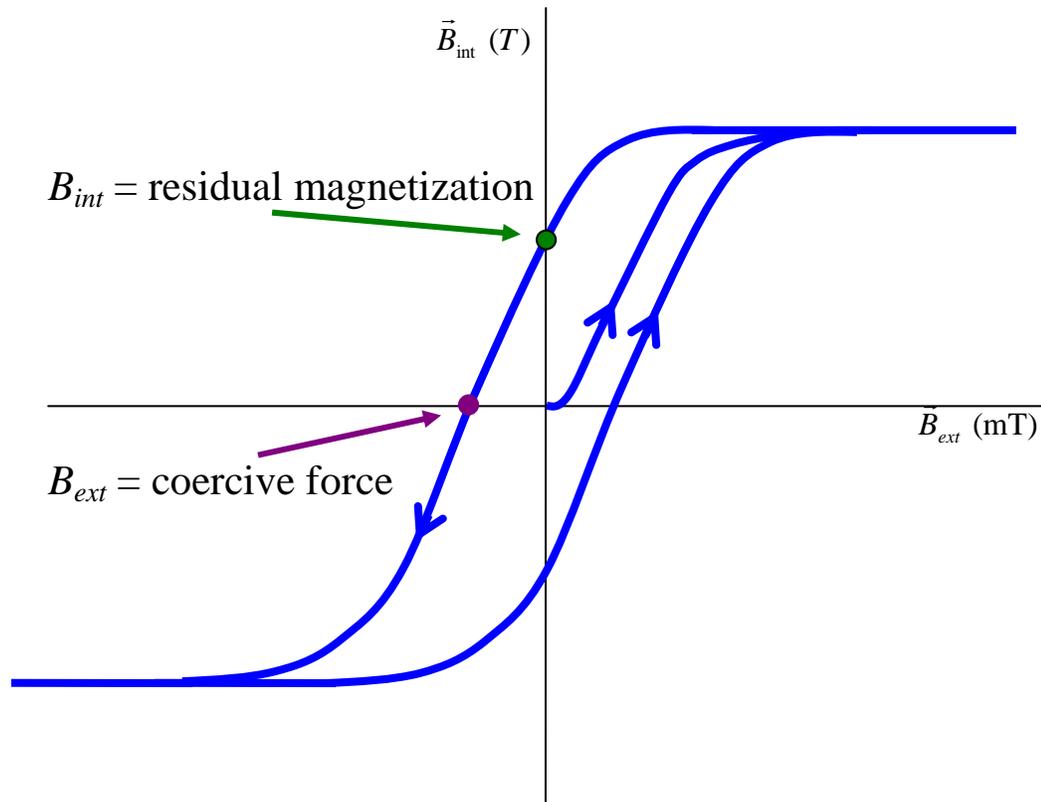
Figure 9.14 A typical hysteresis curve for a permanent magnet with points
showing residual magnetization and coercive force labeled.

However, susceptibilities of ferromagnetic materials are not simple constants as they are
in the paramagnetic and diamagnetic cases. To see why, let us take a piece of unmagnetized
ferromagnetic material. We make a plot of the internal field vs. the external field. Such a plot is
called a "hysteresis curve" as shown in Fig. 9.14. Note that the scales on the two axes are not the
same, as the internal field is several orders of magnitude larger than the external field.

Let's begin with no external field and no internal field, so we start at the origin of the
coordinate system. Then we slowly increase the external field and measure the internal field as
we do so. As the external field increases in the positive direction, the internal field rises rapidly
until all of the domains are aligned with the external field. Once all the domains are aligned, the
internal field can no longer increase. Then let's reduce the external field, which is still positive.
Now the aligned domains do not disalign easily, so the material follows a different curve as the
external field decreases. When the external field goes to zero, there is still an internal field that
remains. This field is called the "residual magnetization." At this point, we reverse the direction
of the external field and increase its magnitude. Eventually, the internal field of the magnet is
forced to zero. The external field required to do this is called, rather inappropriately, the
"coercive force."

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If we continue to increase the external field strength in the negative direction, we begin to magnetize the material in the negative direction, as indicated by the continuation of the hysteresis curve into the fourth quadrant of the figure.

Different ferromagnetic materials have hysteresis curves with different characteristics. A permanent magnet must have a large residual magnetization. The best materials for permanent magnets also have a large coercive force. Soft iron, on the other hand, has large internal fields, but a small residual magnetization and a small coercive force. Thus, soft iron is easy to magnetize in an external field, but it doesn't keep its magnetization once the external field is gone.

Things to remember:

paramagnetic	$\chi = +10^{-5} \, to +10^{-3}$	weakly attracted
diamagnetic	$\chi \approx -10^{-6} \, to -10^{-4}$	weakly repelled
ferromagnetic	$\chi = +10^{+3} \, to +10^{+5}$	strongly attracted

• The relationship between the internal field and external for ferromagnetic materials is described by a hysteresis curve.
• The residual magnetization is the internal field remaining in a magnetic material when the external field is reduced to zero.
• The coercive force is the external field that must be applied to bring the internal field to zero.