
Reading quizzes: no talking, no looking in your books/notes

Q1. T/F: In an interferometer, if you measure the interference pattern as a function of \( \tau \), \( I(\tau) \), you can deduce the power spectrum \( I(\omega) \) of the incident light from your result.

\[ V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}} \]

Q2. "Fringe visibility" refers to
a. closeness/separation of fringe peaks
\( \square \) height/depth of fringe peaks
b. the wavelengths contributing to fringes

Q3. In a typical interferometer, the connection between coherence time and coherence length is:
\( \square \) \( x_c = c \ t_c \)
\( a. \ x_c = c \ t_c \)
\( b. \ x_c = v_{\text{group}} \ t_c \)
\( c. \ x_c = v_{\text{phase}} \ t_c \)
What does intensity do as you send this CW light through interferometer and vary \( \tau \)?

Answer:

\[
I(\tau) = \begin{cases} 
\text{constant} & \text{outside step} \\
\downarrow & \text{inside step}
\end{cases}
\]

\[\tau = 0 \implies \text{maximum}\]

\[|I| : \text{envelope of oscillations}\]

\[I(\tau) \text{ looks very similar} \]

\[I \sim 1 + \Delta(\tau)\]

Coherence time: the amount of delay \( \tau \) needed to cause \( I \) to stop oscillating.

One measure: \( T_c = \int_0^{\infty} |I(\tau)|^2 d\tau \)

The more area, the larger \( T_c \) will be.

The longer you’ll have to wait to have no oscillations.

Coherence length: \( L_c = c T_c \), distance associated w/ \( T_c \). Clear enough.

Fringe visibility:

\[
V = \frac{I_{\text{max}} - I_{\text{min}}}{I_{\text{max}} + I_{\text{min}}}
\]

(or integrated intensity, \( I \)

plitude, \( \Delta I \) pulses)

The problem: \( V \mid |\delta(\tau)| \)

For Gaussian waveform

\[
E(t) = E_0 e^{-\frac{1}{2}(t-\tau)^2}
\]

\[
E(\omega) = E_0 \delta(\omega - \omega_0)
\]

\[
I(\omega) = \frac{1}{2\pi} |E(\omega)|^2 = I_0 e^{-\frac{\omega_0^2}{\omega^2}}
\]

\[
\omega_0 = \frac{c}{\lambda}
\]

\[
I(\omega) \approx I_0 e^{-\frac{(\omega - \omega_0)^2}{\omega_0^2}}
\]

\[
\omega_0 \approx \frac{c}{\lambda}
\]
Can calculate $\gamma(t)$:

$$\gamma(t) = \frac{\int_{-\infty}^{\infty} I(\omega) e^{-i\omega t} d\omega}{\int_{-\infty}^{\infty} I(\omega) d\omega}$$

$$\text{Num} = \int_{-\infty}^{\infty} e^{-\frac{(\omega - \omega_0)^2}{2\Delta^2}} e^{-i\omega t} d\omega = \sqrt{2\pi\Delta^2} e^{-\frac{\omega_0^2}{2\Delta^2}}$$

$$\text{Denom} = \int_{-\infty}^{\infty} e^{-\frac{(\omega - \omega_0)^2}{2\Delta^2}} d\omega = \sqrt{2\pi\Delta^2}$$

$$\gamma = e^{-\frac{\omega_0^2}{2\Delta^2}}$$

I looks like some 2.5 going from 0 to 2$E_0$

$$E_0 = \int_{-\infty}^{\infty} I(\omega) d\omega$$

= total energy (per area) of pulse

What's call $\tau_c$:

$$\tau_c = \int_{-\infty}^{\infty} \left| I(\omega) \right|^2 d\omega$$

$$\tau_c = \frac{1}{\Delta^2}$$

where $\Delta$ was a width of the Gaussian pulse.

$$\tau_c \propto \text{width of pulse}$$
Section 8.5 Fourier Spectroscopy

\[ \text{signal}(t) \sim \left( 2 \int_{-\infty}^{\infty} I(t) e^{i \omega t} dt \right) \left[ 1 + \text{Re}(\chi(t)) \right] = 2 \varepsilon_0 \left( 1 + \frac{\text{Re}(\chi(t))}{\varepsilon_0} \right) \]

If we measure \[ \text{signal} \]

...can we deduce the frequency spectrum?

Yes:

\[ \text{NS}(\omega) = \frac{\text{signal}}{\text{propagation}} = 2 \varepsilon_0 + 2 \text{Re} \int I(\omega) e^{-i \omega t} dt \]

Take FT of \[ \text{NS}(\omega) \] :

\[ \text{FT} (\text{NS}(\omega)) = \text{FT} (2 \varepsilon_0) + \text{FT} \left( 2 \text{Re} \left( I(\omega) e^{-i \omega t} dt \right) \right) \]

\[ = 2 \varepsilon_0 \frac{\text{FT}(\text{I}(\omega))}{\sqrt{2\pi}} \]

\[ \text{FT} \left( 1 \text{FT} (I(\omega)) + 1 \text{FT} (I(-\omega)) \right) \]

\[ = 2 \varepsilon_0 \frac{\varepsilon_0 \delta(\omega)}{\sqrt{2\pi}} + \text{FT} (I(\omega)) + \text{FT} (I(-\omega)) \]

\[ \text{FT} (\text{NS}(\omega)) \]

\[ = 2 \varepsilon_0 \delta(\omega) + \text{FT} (I(\omega)) + \text{FT} (I(-\omega)) \]

Typically, \[ \text{I}(\omega) \] is complex.

Expected graph

\[ \text{I}(\omega) \text{ identified!} \]
Lecture 26: Fri, 7 Mar 2008

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Q1. In a "Young's experiment setup, the light will produce better fringes if it comes from:
   a. a small source
   b. a large source
   c. doesn't depend on source size

Q2. Young's experiment produces interference between how many slits?
   a. 1
   b. 2
   c. 3
   d. More than 3

Q3. When did Young live?
   a. Before Michelson
   b. Contemporary with Michelson
   c. After Michelson
Young's 2-slit experiment

- First conclusive demo that light = wave

Quasi-mechanistic light

Like Michelson; 2 beams of light travel different distances

\[ I = 2I_0 \left( 1 + \cos k(d_0 - d_1) \right) \]

if not completely symmetric, 2 different phases at each slit,

\[ I = 2I_0 \left( 1 + \cos \left( k(d_0 - d_1) + \phi_0 \right) \right) \]

Some geometry

\[ \delta_2 - \delta_1 = D \left( 1 + \frac{1}{2} \left( \frac{y^2}{D} \right)^2 \right) - D \left( 1 + \frac{1}{2} \left( \frac{y^2}{D} \right)^2 \right) \]

\[ = \frac{1}{2D} \left[ y^2 - y h + \frac{h^2}{4} - y^2 + y h - \frac{h^2}{4} \right] \]

\[ = \frac{y h}{D} \]

\[ I = 2I_0 \left( 1 + \cos \left( \frac{y h}{D} + \phi_0 \right) \right) \]
What if a finite source?

Not just a simple convolution of
phase of source is varying (random?)

Treat the extended source as a series of point sources, each with slightly different phase $\phi_j$

For each $j$, $E_j = |E_j| \left[ e^{-ik(r_j + d_j) \cdot \hat{t} + \phi_j} + i k (r_j + d_j) \cdot \hat{t} + \phi_j \right]

$$E_{tot} = \sum_j E_j$$

$$I \propto |E| \cdot |E| = \left| \sum_j E_j \right|^2 = \sum_{j,m} |E_j| \cdot |E_m| \exp[i(k(r_{jm} + d_j) - \phi_m)]$$

Assume a noncoherent source

1. LED, starlight or lightbulb, not laser

$\phi_j - \phi_m$ causes oscillations which average out unless $j = m$

$$|E_{tot}|^2 = \sum_j |E_j|^2 \cdot |E_j|^2 \cdot 2 \Re \left[ \exp \left[ -ik \cdot d_j \cdot \hat{t} + \phi_j \right] \right]$$

$\phi_j - \phi_m$ is distance dependent for the $j$ vs. $m$ sources
\[
\begin{align*}
\frac{\epsilon_{\text{mm}}}{2} = & \sum_j \left( \epsilon_j \right)^2 + \sum_j \epsilon_j \left( \sum_j \left| \epsilon_j \right|^2 \right) e^{-i \left( k_j \frac{\epsilon_j}{2} + k_j \frac{\epsilon_j}{2} \right)} \\
\text{part in multiplicative constant} & \quad \text{invariant of \( \epsilon \)} \\
I = & \quad 2 \sum_j \epsilon_j^2 + 2 \sum_j \epsilon_j \epsilon_j^* e^{-i k_j \frac{\epsilon_j}{\hbar}} \sum_j \epsilon_j^* e^{i k_j \frac{\epsilon_j}{\hbar}} \\
I^{(h)} = & \left( 2 \sum_j \epsilon_j \right) \left\{ 1 + e^{-i \frac{k_0}{\hbar} \sum_j \epsilon_j \epsilon_j^* e^{i k_j \frac{\epsilon_j}{\hbar}}} \right\} \\
\gamma(h) = & \text{"degree of coherence"}
\end{align*}
\]

Looks a lot like our time coherence formula:

\[
\text{Recall} \quad \gamma(p) = \frac{\int I^{(p)} e^{-i\omega_0 t} dt}{\int I^{(p)} dt}
\]

Main difference \( \gamma(t) \) had continuous integral
- \( \gamma(h) \) has \( \sum \) factors
- \( \gamma(h) \) has \( e^{-i \hbar \omega} \) factor
- \( \gamma(h) \) defines position of fringe on screen

Notes:
1) When \( h \) large, it's like when \( t \) was large
   \( \delta \) amplitude of \( \gamma \approx 0 \), no oscillations visible
2) Like before \( V = |\gamma| \)
3) Like coherence time

\[
h_c = \int_0^\infty |\gamma(h)|^2 dh \text{ sets boundary before large } h \text{ (no oscillations)}
\]

\( h_c \to \gamma \text{ (oscillations)"}

That's all for now!