TE Modes of a Rectangular Waveguide

by Dr. Colton, Physics 442 (last updated: Winter 2020)

Calculating TE Modes

Using Mathematica, we can calculate the first 15 TE modes for a rectangular waveguide. I’m using dimensions of \( a = 10 \) cm and \( b = 7 \) cm, which were chosen arbitrarily.

Here are the cutoff frequencies of the first 15 modes (ignore the 0 frequency); they are shown first in table form and then in list form in ascending order.

\[
\begin{align*}
    &0, \\
    &9.42478 \times 10^9, 1.3464 \times 10^{10}, 2.69279 \times 10^{10}, 4.03919 \times 10^{10} \\
    &1.88496 \times 10^{10}, 3.13164 \times 10^{10}, 3.90455 \times 10^{10}, 4.93046 \times 10^{10} \\
\end{align*}
\]

\[
\]

The \( k(\omega) \) dispersion relations for the first 15 modes are as follows:

\[
\begin{align*}
    &\sqrt{3 \times 8}; \\
    \omega[m, n] := 3 \times 8 \sqrt{(m \pi / a)^2 + (n \pi / b)^2} \\
    \text{cutofftable} = \text{Table}[\omega[m, n], \{m, 0, 3\}, \{n, 0, 3\}]; \\
    \text{cutofftable} // \text{MatrixForm} \\
\end{align*}
\]

\[
\begin{align*}
    \sqrt{-298.96}, \omega & \quad \sqrt{-3947.84}, \omega & \quad \sqrt{-8882.64}, \omega \\
    \omega & \quad \sqrt{-12004.7}, \omega & \quad \sqrt{-19114.8}, \omega \\
\end{align*}
\]
**$k(\omega)$ dispersion relation plots**

For a given mode its dispersion relation is set by one of the following curves.

![dispersion relation plots](image)

- This is the 11 mode
- This is the 01 mode
- This is the 10 mode

For a given mode its dispersion relation is set by one of the following curves.

- no modes possible here
- only one mode possible here
Recall that the governing field for the TE modes is the $z$ component of the magnetic field (because the electric field has no $z$-component). Here are plots of $B_z$ for the first 15 modes (ignore the upper left one). Tannish white is the positive antinode and blue is the negative antinode.

$E_z = 0$ by definition, and all of the other nonzero components of the fields, namely $E_x, E_y, B_x$, and $B_y$, can be calculated from $B_z$. 