Fall 2016  
Physics 441  
Exam 2  
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No time limit. Student calculators are allowed. One page of handwritten notes allowed (front & back). Books not allowed. A HANDOUT WITH FRONT AND BACK INSIDE COVERS OF GRIFFITHS TEXTBOOK SHOULD BE PROVIDED. If not, please ask the Testing Center for it and/or have them call me.

Instructions: Please label & circle/hide your answers. Show your work, where appropriate! And remember: in any problems involving Gauss’s Law, you should explicitly show your Gaussian surface. For all problems, unless otherwise specified you may assume that you are dealing with electrostatics, i.e. the charges are not moving and the fields have come to equilibrium.

Integral/derivative table: One or more of the following integrals or derivatives may or may not be helpful on the exam. If you find yourself needing anything more complicated than this, then you have likely made an error.

\[(\theta_x = \text{Mathematica for } \frac{d}{dx})\]

\[
\begin{align*}
\text{In[1]} &= \int e^{ax} \, dx \\
\text{Out[1]} &= e^{ax}/a \\
\text{In[2]} &= \int x e^{ax} \, dx \quad // \quad \text{Expand} \\
\text{Out[2]} &= \frac{x e^{ax}}{a} + \frac{e^{ax}}{a} \\
\text{In[3]} &= \int x^2 e^{ax} \, dx \quad // \quad \text{Expand} \\
\text{Out[3]} &= \frac{2 e^{ax}}{a} - \frac{2 a x^2 e^{ax}}{a^3} + \frac{e^{ax} x^2}{a} \\
\text{In[4]} &= \theta_x \cdot e^{ax} \\
\text{Out[4]} &= a e^{ax} \\
\text{In[5]} &= \theta_x (x e^{ax}) \\
\text{Out[5]} &= \psi^{\star x} + \psi \psi^{\star x} \\
\text{In[6]} &= \theta_x (x^2 e^{ax}) \\
\text{Out[6]} &= 2 e^{ax} x + a e^{ax} x^2
\end{align*}

Some Legendre polynomials:

\[
\begin{align*}
P_0(x) &= 1 \\
P_1(x) &= x \\
P_2(x) &= -3/2 x^2 - 1/2 \\
P_3(x) &= 5/2 x^3 - 3/2 x
\end{align*}
\]

Orthogonality of the Legendre polynomials:

\[
\int_1^3 P_\ell(x) P_m(x) \, dx = \begin{cases} 
0 & \text{if } \ell \neq m \\
\frac{2}{2\ell + 1} & \text{if } \ell = m
\end{cases}
\]

\[
\int_0^\pi P_\ell(\cos \theta) P_m(\cos \theta) \sin \theta \, d\theta = \begin{cases} 
0 & \text{if } \ell \neq m \\
\frac{2}{2\ell + 1} & \text{if } \ell = m
\end{cases}
\]

Orthogonality of the sine functions:

\[
\int_0^1 \sin(mx) \sin(nx) \, dx = \begin{cases} 
0 & \text{if } n \neq m \\
1/2 & \text{if } n = m
\end{cases}
\]

\[
\int_0^{2\pi} \sin(mx) \sin(nx) \, dx = \begin{cases} 
0 & \text{if } n \neq m \\
1/2 & \text{if } n = m
\end{cases}
\]

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1.3. An infinitely long rectangular metal pipe (sides a and b) is grounded, but one end, at \( x = 0 \), is maintained at a specified potential \( V_0 \). See the figure.

The potential can be described through the general equation:

\[
V(x, y, z) = \left( A e^{\sqrt{k^2 + 1}x} + B e^{-\sqrt{k^2 + 1}x} \right) (C \sin k y + D \cos k y)(E \sin k z + F \cos k z)
\]

Which of the following is true about the constants in the equation?

\[
\sqrt{k} = \beta \quad A = 0 \quad B = 0 \quad C = 0 \quad D = 0 \quad E = 0 \quad F = 0 \quad k = \frac{n\pi}{a} \quad (\text{where} \ n = \text{an integer})
\]

(a) I only
(b) I and II only
(c) I and III only
(d) I and IV only
(e) II and III only
(f) II and IV only
(g) III and IV only
(h) I, II, and III only
(i) I, II, and IV only
(j) II, III, and IV only
(k) II, III, and IV only
(l) I, II, III, and IV

1.4. Points A and B are the same large distance from an electric dipole, but in different directions as per the figure. The dipole is depicted by the arrow (dipole moment in the direction of the arrow). What is true of the magnitude of the E-field at point A compared to point B?

(a) \( |\vec{E}_a| = 0 \)
(b) \( |\vec{E}_b| = 0 \)
(c) \( |\vec{E}_a| \text{ and } |\vec{E}_b| \text{ both } = 0 \)
(d) \( |\vec{E}_a| > |\vec{E}_b| \) (and \( |\vec{E}_a| \neq 0 \))
(e) \( |\vec{E}_a| < |\vec{E}_b| \) (and \( |\vec{E}_a| \neq 0 \))
(f) \( |\vec{E}_a| = |\vec{E}_b| \) (and neither is zero)

\[
\vec{E} = \frac{q}{4\pi\varepsilon_0} \frac{q_0 r}{r^3} \left( 2(x_0 r \hat{x} + y_0 r \hat{y} + z_0 r \hat{z}) \right)
\]

\[
\vec{A} : \vec{E} \cdot \vec{B} = 0 \quad \vec{E} \cdot \vec{B} = 0
\]

\[
\vec{B} = \frac{q}{4\pi\varepsilon_0} \frac{q_0 r}{r^3} \left( 2(x_0 r \hat{x} + y_0 r \hat{y} + z_0 r \hat{z}) \right)
\]

\[
\vec{B} = \frac{q}{4\pi\varepsilon_0} \frac{q_0 r}{r^3} \left( 2(x_0 r \hat{x} + y_0 r \hat{y} + z_0 r \hat{z}) \right)
\]
(14 pts) Problem 2: A thin insulating spherical shell (radius R) has a uniform surface charge density \( \sigma_0 \) coating it. The center of the sphere is fixed at a distance \( d \) above the center of an infinite, grounded conducting plate.

(a) Find the potential \( V \) for an arbitrary point \((x, y, z)\) that lies inside the spherical shell. Use the specified origin of coordinates.

From knowledge of Gauss's law, \( V \) from \( +\sigma_0 \) is constant (since \( \varepsilon \) is infinite)

\[
V \text{ from } -\sigma_0 \text{ is same as from a point charge at } (0, 0, -d)
\]

The resultant \( V \) is same as from point charge, \( V = \frac{q}{4\pi\varepsilon_0 r} \)

\[
V(x, y, z) = \frac{\sigma_0}{4\pi\varepsilon_0} \frac{1}{R} - \frac{\sigma_0}{4\pi\varepsilon_0} \frac{1}{\sqrt{x^2 + y^2 + (z+d)^2}}
\]

for points inside the spherical shell

Could simplify if desired:

\[
V = \frac{\sigma_0 R}{\varepsilon_0} \left( 1 - \frac{R}{\sqrt{x^2 + y^2 + (z+d)^2}} \right)
\]

(b) Sketch the electric field lines on the figure above. Or, you can reproduce the figure and sketch here.

\( \text{like the upper half of a dipole's field lines} \)
(16 pts) Problem 4: A sphere of radius $R$ centered on the origin carries a surface charge density of $\sigma \cos \theta$. Find the approximate potential and the field for this charge distribution for points along the y-axis $(0,0,z)$, where $|y| \gg R$.

\[
\begin{align*}
\varphi &= \int g(r) \cdot \hat{e}_r \, dr \quad \text{I'll just keep the } \theta \text{-component of this.} \\
\varphi &= \int \left( \sigma \cos \theta \right) \left( \hat{e}_\theta \cdot \hat{e}_r \right) \, R^2 \sin \theta \, d\theta \, dr \\
&= 2\pi \sigma_0 R^2 \frac{2}{3} \int_0^\pi \cos^2 \theta \sin \theta \, d\theta \\
&= 2\pi \sigma_0 R^2 \frac{2}{3} \left[ \frac{1}{3} \cos^3 \theta \right]_0^\pi \\
&= \frac{2}{3} \sigma_0 R^2 \\
\end{align*}
\]

\[
V_{\text{on the}} = \frac{1}{\sqrt{|y|}} \quad \text{for points on the y-axis } \theta = \frac{\pi}{2} \\
V = 0 \\
\theta = 0 \Rightarrow \sqrt{|y|} \\
\text{Symmetry: } V = 0 \quad \text{everywhere in the x-z plane.} \\
\]

\[
E_{\text{on the}} = \frac{\sigma_0 R^2}{|y|} \left( 2 \cos^3 \theta - 3 \sin \theta \right) \\
\]

\[
\begin{align*}
E &= \frac{\sigma_0 R^2}{|y|^\frac{3}{2}} \left( \frac{1}{|y|} \left( 2 \cos^3 \theta - 3 \sin \theta \right) \right) \\
E &= \frac{\sigma_0 R^2}{3|y|^\frac{3}{2}} \left( -\frac{2}{3} \right) \\
\end{align*}
\]

\[
\left( \text{for points on the y-axis, with } y \gg R \right. \\
\left. \text{the negative y-axis, the field is still in the } -\hat{z} \text{ direction.} \right)
\]

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Problem 6. A point charge \( q \) is at the origin. It is surrounded by a spherical dielectric shell (dielectric constant \( \varepsilon_r \)) with inner radius \( a \) and outer radius \( b \). (a) Determine \( \mathbf{D} \) in the three regions: (i) \( r < a \), (ii) \( a < r < b \), and (iii) \( r > b \). (b) Determine \( \mathbf{E} \) in the same three regions.

(a) For all three regions, we can use a Gaussian surface like this, and if \( q \) within, \( \mathbf{D} = \mathbf{q}/4 \pi e_0 \) everywhere.

\[
\mathbf{D} = \frac{q}{4 \pi e_0} \mathbf{r}
\]

(b) \( \mathbf{D} = \varepsilon_0 \varepsilon_r \mathbf{E} \rightarrow \mathbf{E} = \frac{\mathbf{D}}{\varepsilon_0 \varepsilon_r} \) where \( \varepsilon_0 \varepsilon_r \) is the dielectric constant for the particular region.

Regions (i) and (iii): \( \varepsilon_r = 1 \)

\[
\mathbf{E} = \frac{\mathbf{D}}{\varepsilon_0} \quad \text{for all \( r \)}
\]

Region (ii): \( \varepsilon_r > \varepsilon_0 \)

\[
\mathbf{E} = \frac{\mathbf{D}}{\varepsilon_0 \varepsilon_r} \quad \text{for all \( r \)}
\]