Lesson 13
Applications of Time-varying Circuits

Class 38
Today we will:
• find out how transformers work
• learn about how electrical power is generated and delivered to our homes.

The Series LRC Circuit
• Be able to draw the impedance diagram and find the magnitude and phase angle of the impedance

Resonance
• Resonance is where the inductive and capacitive reactances are equal.
• The resonant frequency is:
  \[ X_L = X_C \Rightarrow \omega = \frac{1}{\sqrt{LC}} \]
• The impedance is minimum and the current is maximum.
  \[ Z = \sqrt{(X_L - X_C)^2 + R^2} = R \]

Transformers
Iron Core Inductors

Adding an iron core to an inductor accomplishes two things:
• It increases the magnetic field
• It tends to keep the magnetic field confined in the core.

Iron Core Inductors

Note how an iron core modifies the magnetic field lines of a wire coil.

Iron Core Inductors

• We can even make an iron core that forms a closed loop.

Iron Core Inductors

• We can use Ampère’s Law around one field line to find the magnetic field. Assuming $B$ is uniform:
  \[ \Lambda_B = \mu_0 I_{\text{enc}} \]
  \[ B\ell = \mu_0 N_i i \]
  \[ B = \frac{\mu_0 N_i i}{\ell} \]

Iron Core Inductors

• We can use Faraday’s Law to find the impedance:
  For 1 loop: $\Phi_B = AB = A\mu_0 n_i = A\mu_0 \frac{N_i}{\ell} i$
  \[ V_i = -N_i \frac{d\Phi_B}{dt} = -N_i A \frac{\mu_0 N_i}{\ell} \frac{di}{dt} = -L \frac{di}{dt} \]
  \[ \Rightarrow L = \frac{\mu_0 N_i^2 A}{\ell} \]

Iron Core Inductors

Mutual Inductance

• We can also put two coils on the same core or yoke.
Mutual Inductance

• We attach a power supply to one coil. This is the “primary.”

Mutual Inductance

• Since the magnetic flux in the upper coil changes in time, an EMF is induced.

Transformers

• This is called a transformer.

Transformers

• We attach a load to the other coil. This coil is the “secondary.”

Transformers

• The magnetic flux through one coil of either winding is the same, as the number of filed lines is the same.

Transformers

• Since the flux is the same through both coils, the change in flux is also the same:

\[
V_1 = -N_1 \frac{d\Phi_s}{dt}
\]

\[
- \frac{d\Phi_s}{dt} = \frac{V_1}{N_1} = \frac{V_i}{N_i}
\]

\[
\Rightarrow \frac{V_1}{V_i} = \frac{N_i}{N_1}
\]
Transformers

• If there are 10 times as many windings in the secondary as the primary, there is 10 times the voltage in the secondary. This is called a “step-up” transformer.
• If there are 10 times fewer windings in the secondary as the primary, there is 10 times less voltage in the secondary. This is called a “step-down” transformer.

Power in AC Circuits

Recall that the power provided by a power supply is

\[ P = i_{rms}\varepsilon_{rms}\cos\phi. \]

If the load is resistive, the phase angle is zero and

\[ P = i_{rms}^2\varepsilon_{rms}. \]

The power dissipated in a resistor is

\[ P = i_{rms}^2V_{rms}. \]

Power and Transformers

• Transformers have very little power loss to heating, etc.
• The power provided by the primary is used in the secondary.
• If the power factors are approximately equal to 1:

\[ i_1V_1 = i_2V_2 \]

Power Transmission

• We can model a transmission line as a simple circuit.

Transmission Lines

\[ V \]

\[ \text{Line} \]

\[ \text{R}_1 \text{ Load resistance} \]

\[ \text{R}_1 \text{ resistance} \]
Transmission Lines

\[ I = \frac{V}{R_L + R_t} \]

\[ V_L = I R_L, \quad V_t = I R_t \]

\[ P_L = I V_L, \quad P_t = I V_t, \quad P_L = P_t \]

Let’s compare two cases with a 250 W load and a 10Ω transmission line:

<table>
<thead>
<tr>
<th>Voltage of the power source</th>
<th>100V</th>
<th>250V</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance of the load</td>
<td>10 Ω</td>
<td>229.6 Ω</td>
</tr>
<tr>
<td>Power loss in the load</td>
<td>250 W</td>
<td>250 W</td>
</tr>
<tr>
<td>Power loss in the line</td>
<td>250 W</td>
<td>10.9 W</td>
</tr>
<tr>
<td>Power provided by the battery</td>
<td>500 W</td>
<td>260.9 W</td>
</tr>
</tbody>
</table>

Transmission Lines

• Conclusion: Transmission lines are more efficient when they have very high voltages.
• Major lines have voltages of several hundred kV.
• Substations lower the voltage of local lines to 4-8 kV.

Transmission

Lines into a Home

• If the primary voltage is 2400 V, then the local transformer has a 10:1 ratio of turns.
• The middle of the secondary coil is attached by a wire to ground.
• A ground wire and wires from the two ends of the secondary come into your home.

• The ground wire is at 0 V, and the other two wires at 120 V (rms).
• The 120 V wires are out of phase with respect to each other.
The Service Panel

- The service panel is where outside power comes in and wires are then distributed through different circuits throughout your house.
- Either 120 V or 240 V circuits can be taken from the service panel.
- The service panel is often called the “circuit breaker box.”

Circuit Breakers

Circuit breakers provide two functions:
- They serve as switches to shut off power to parts of your house.
- They automatically shut of power if too much current flows into the circuit.
- Large currents cause wires to heat and start fires.
Circuit Breakers

• A circuit breaker also contains a solenoid that controls a second switch.
• When the current rises above a given level, the solenoid opens the circuit in a fraction of a second.
• The circuit breaker switch flips to a middle position between on and off and can be reset by turning the switch back on.

Class 39

Today we will:
• learn about wires used in homes
• learn how switches and outlets are wired
• learn how to wire a 3-way switch
• find out about safety devices: grounds, GFCI's, and AFCI's

Wires

• Wires are bundled into cables of three or four wires.
• Conductors are either copper or aluminum.
• Copper is a better conductor, more flexible, and corrodes less.
• Aluminum is cheaper.
• Special components are made for aluminum wires.

Wires and Heat – E&M

• The source of heat is resistance in the wire. A length of wire generates heat at the rate:

\[
P = \frac{\Delta Q}{\Delta t} = IV = I^2 R = I^2 \rho \frac{L}{\pi r^2}
\]
Wires and Heat – E&M

- The source of heat is resistance in the wire. A length of wire generates heat at the rate:

\[ P = \frac{\Delta Q}{\Delta t} = IV = I^2R = I^2\rho \frac{L}{\pi r^2} \]

- The more current in a circuit, the larger the wire must be to keep the wire from overheating.

Wires

Rough rule of thumb:
- Cu can take 4 A/mm²
- Al can take 2.3 A/mm²

Copper Wires

<table>
<thead>
<tr>
<th>AWG</th>
<th>Current (A/mm²)</th>
<th>Diameter (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>10</td>
<td>0.50</td>
<td>4.0</td>
</tr>
<tr>
<td>12</td>
<td>0.63</td>
<td>3.5</td>
</tr>
<tr>
<td>14</td>
<td>0.85</td>
<td>3.0</td>
</tr>
<tr>
<td>16</td>
<td>1.05</td>
<td>2.8</td>
</tr>
<tr>
<td>18</td>
<td>1.09</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Table 13.2 Data for common copper wire sizes

Wiring Switches and Outlets

Switches

- Single-pole single-throw
- Double-pole single-throw
- Single-pole double-throw
- Double-pole double-throw

Switches are placed along the hot wire.
Switches

If two switches control the same light, double throw switches are used.

3-Way Switches

down up

down

3-Way Switches

down up

3-Way Switches

up up
Parallel Outlets

Outlets

• Series outlets are easier to wire.
• However, if the connection to one series outlet is bad, the connection affects all downstream outlets.

Safety Devices

No Ground
Faulty wiring causes the outside of a toaster to have 120V on it. Current flows through you.

Ground
If the toaster is grounded, current flows through the ground wire.

Ground
Think of the toaster as a battery and you and the ground wire as two resistors in parallel.
Ground

- Two resistors in parallel

GFCI

- Ground Fault Circuit Interrupter
- Shuts off power when current in the hot wire is different than current in the neutral wire.
- Makes use of a differential transformer.
- Used in kitchens and bathrooms.
- Built into outlets.

GFCI Outlets

- GFCI outlets are better wired in series, as the GFCI works for all downstream outlets.

Differential Transformer

- If the current in the hot wire is the same as the current in the neutral wire, the induced current in the secondary is zero.

- Solenoid switch opens if some current is lost because of a grounding problem.

AFCI

- Arc Fault Circuit Interrupter
- Shuts off power when there is arcing between hot wire and ground or neutral wires.
- Used in bedrooms.
- Built into circuit breaker.
- When arcing occurs, spikes, squared waves, etc., are typical. Various methods of detection are used.
Class 40

Today we will:
• review basic characteristics of waves
• introduce definitions of wave terminology
• show how Maxwell’s Equations predict electromagnetic waves
• discuss the spectrum of electromagnetic radiation
• learn how radio antennas send and receive signals

Wave Review
• Sine wave \( y(x,t) = A \sin(kx - \omega t) \) at \( t=0 \).

Wave Review
• What do the parameters mean?

\( y(x,t) = A \sin(kx - \omega t) \)

• “Snapshot” \( t=0 \)

\( A \) is the amplitude.

Wave Review
• What do the parameters mean?

\( y(x,t) = A \sin(kx - \omega t) \)

• “Snapshot” \( t=0 \)

\( \Delta x = \lambda \)

\( \lambda \) is the wavelength.
### Wave Review

**What do the parameters mean?**

\[ y(x, t) = A \sin(kx - \omega t) \]

- "Snapshot" \( t = 0 \)
  
  \[ \Delta x = \lambda \]

  \[ k = \frac{2\pi}{\lambda} \]

  \( k \) is the wavenumber = # of radians in 1 meter

**"Oscilloscope trace"** \( x = 0 \)

\[ y(x, t) = A \sin(kx - \omega t) \]

- \( \Delta \theta = k \Delta x = 2\pi \)
  
  \[ \omega = \frac{2\pi}{T} = 2\pi f \]

- \( \Delta \theta = \omega \Delta t = 2\pi \)

\( T \) is the period.

**Translation**

- To translate a general function to the right 3 units:
  
  \[ y(x, t) = A \sin(kx - \omega t) \]

- To make the function move to the right at a speed \( v \):
  
  \[ y(x, t) = A \sin\left(k \left( x - \frac{\omega t}{k} \right)\right) \]

  \[ \Rightarrow v = \frac{\omega}{k} \]
Wave Velocity

\[ v = \frac{\omega}{k} = 2\pi f \frac{\lambda}{2\pi} \]

\[ v = \lambda f \]

Wave Review

• Sine wave \( y(x,t) = 3\sin(4x - 5t) \)
• Amplitude \( A=3 \)
• Wavenumber \( k=4 \)
• Angular frequency \( \omega=5 \)
Wave Review

• Sine wave \( y(x,t) = 3\sin(4x - 5t) \)

• Amplitude \( A=3 \)
• Wavenumber \( k=4 \)
• Angular frequency \( \omega=5 \)
• Wavelength \( \lambda = \frac{2\pi}{k} = \frac{2\pi}{4} \)
• Frequency \( f = \frac{\omega}{2\pi} = \frac{5}{2\pi} \)
• Period \( T = \frac{1}{f} = \frac{2\pi}{5} \)
• Velocity \( v = +\lambda f = +\frac{\pi}{2} \frac{5}{2\pi} = +\frac{5}{4} \)

Electromagnetic Radiation
What We Know about Radiation

- The electric and magnetic fields are perpendicular.
- The direction of motion is perpendicular to both E and B. \( \hat{E} \times \hat{B} = \hat{v} \)
- The magnitude of the magnetic field is \( \frac{1}{c} \) times smaller than the electric field.

We Guess a Solution

- Assume we have an electromagnetic wave that moves in the \( x \) direction.

\[
\hat{E}(x,t) = E_0 \sin(kx - \omega t) \hat{y} \\
\hat{B}(x,t) = \frac{E_0}{c} \sin(kx - \omega t) \hat{z}
\]

Maxwell’s Equations

- Gauss’s Law of Electricity
  \[ \nabla \cdot \hat{E} = \frac{\rho}{\varepsilon_0} \]
- Gauss’s Law of Magnetism
  \[ \nabla \cdot \hat{B} = 0 \]
- Ampere’s Law
  \[ \nabla \times \hat{B} = \mu_0 \left( j + \varepsilon_0 \frac{\partial \hat{E}}{\partial t} \right) \]
- Faraday’s Law
  \[ \nabla \times \hat{E} = -\frac{\partial \hat{B}}{\partial t} \]

Maxwell’s Equations in Empty Space

- Gauss’s Law of Electricity
  \[ \nabla \cdot \hat{E} = 0 \]
- Gauss’s Law of Magnetism
  \[ \nabla \cdot \hat{B} = 0 \]
- Ampere’s Law
  \[ \nabla \times \hat{B} = \mu_0 \varepsilon_0 \frac{\partial \hat{E}}{\partial t} \]
- Faraday’s Law
  \[ \nabla \times \hat{E} = -\frac{\partial \hat{B}}{\partial t} \]

Gauss’s Law of Electricity

\[ \nabla \cdot \hat{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z} \]
\[ \hat{E}(x,t) = E_0 \sin(kx - \omega t) \hat{y} \]
\[ \nabla \cdot \hat{E} = \frac{\partial E_y}{\partial y} = 0 \]

Gauss’s Law of Magnetism

\[ \nabla \cdot \hat{B} = \frac{\partial B_x}{\partial x} + \frac{\partial B_y}{\partial y} + \frac{\partial B_z}{\partial z} \]
\[ \hat{B}(x,t) = \frac{E_0}{c} \sin(kx - \omega t) \hat{z} \]
\[ \nabla \cdot \hat{B} = \frac{\partial B_z}{\partial z} = 0 \]
Faraday's Law

\[ \nabla \times \mathbf{E} = \left[ \frac{\partial E_z}{\partial y} - \frac{\partial E_y}{\partial z} \right] \hat{z} + \left[ \frac{\partial E_x}{\partial z} - \frac{\partial E_z}{\partial x} \right] \hat{x} + \left[ \frac{\partial E_y}{\partial x} - \frac{\partial E_x}{\partial y} \right] \hat{y} \]

\[ \mathbf{B}(x, t) = -\frac{E_0}{c} \sin(kx - \omega t) \hat{z} \]

\[ \mathbf{E}(x, t) = E_0 \sin(kx - \omega t) \hat{y} \]

\[ \nabla \times \mathbf{E} = \left[ \frac{\partial E_z}{\partial y} + \frac{\partial E_y}{\partial z} \right] \hat{z} = \frac{E_0 k \cos(kx - \omega t)}{c} \]

\[ \frac{\partial \mathbf{B}}{\partial t} = -\frac{\mathbf{E}}{c \omega} \cos(kx - \omega t) \]

\[ \frac{\partial \mathbf{B}}{\partial t} = \frac{E_0}{c} k \cos(kx - \omega t) \]

\[ \frac{\partial \mathbf{E}}{\partial t} = \frac{E_0}{c} k \cos(kx - \omega t) \]

\[ \nabla \times \mathbf{B} = \left[ \frac{\partial B_y}{\partial z} - \frac{\partial B_z}{\partial y} \right] \hat{z} + \left[ \frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z} \right] \hat{x} + \left[ \frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x} \right] \hat{y} \]

Ampere's Law

\[ \nabla \times \mathbf{B} = \left[ \frac{\partial B_y}{\partial z} \right] \hat{z} + \left[ \frac{\partial B_z}{\partial x} \right] \hat{x} + \left[ \frac{\partial B_x}{\partial y} \right] \hat{y} \]

\[ \mathbf{B}(x, t) = \frac{E_0}{c} \sin(kx - \omega t) \hat{z} \]

\[ \mathbf{E}(x, t) = E_0 \sin(kx - \omega t) \hat{y} \]

\[ \nabla \times \mathbf{B} = \left[ \frac{\partial B_y}{\partial z} \right] \hat{z} = -\frac{E_0}{c} k \cos(kx - \omega t) \]

\[ \frac{\partial \mathbf{B}}{\partial t} = -\frac{\mathbf{E}}{c \omega} \cos(kx - \omega t) \]

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Combining Ampere's Law and Faraday's Law

\[ \nabla \times \mathbf{B} = \left[ \frac{\partial B_y}{\partial z} - \frac{\partial B_z}{\partial y} \right] \hat{z} = \left[ \frac{\partial B_z}{\partial x} - \frac{\partial B_x}{\partial z} \right] \hat{x} + \left[ \frac{\partial B_x}{\partial y} - \frac{\partial B_y}{\partial x} \right] \hat{y} \]

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Wave Equation

\[ \frac{\partial^2 \mathbf{B}}{\partial t^2} = \frac{\mu_0 \varepsilon_0}{c^2} \frac{\partial^2 \mathbf{E}}{\partial x^2} \]

\[ \mathbf{B}_0 = \frac{E_0}{c} \sin(kx - \omega t) \]

\[ -\frac{E_0}{c} [k^2 \sin(kx - \omega t)] = -\mu_0 \varepsilon_0 \left[ \frac{\omega^2}{c^2} \sin(kx - \omega t) \right] \]

\[ \frac{\omega^2}{c^2} = \frac{1}{\mu_0 \varepsilon_0} \]

\[ c = \frac{1}{\sqrt{\mu_0 \varepsilon_0}} \]
Electromagnetic Spectrum

<table>
<thead>
<tr>
<th>Name</th>
<th>Typical Source</th>
<th>Approximate Wavelength</th>
</tr>
</thead>
<tbody>
<tr>
<td>Radio</td>
<td>Oscillating circuits</td>
<td>&gt;10 cm</td>
</tr>
<tr>
<td>Microwave</td>
<td>Electronic devices</td>
<td>100 μm – 10 cm</td>
</tr>
<tr>
<td>Infrared</td>
<td>Atoms, molecules</td>
<td>700 nm – 100 μm</td>
</tr>
<tr>
<td>Visible Light</td>
<td>Atoms</td>
<td>400 – 700 nm</td>
</tr>
<tr>
<td>Ultraviolet</td>
<td>Atoms</td>
<td>1 – 400 nm</td>
</tr>
<tr>
<td>X-rays</td>
<td>Inner shells of atoms</td>
<td>1 pm – 1 nm</td>
</tr>
<tr>
<td>Gamma-rays</td>
<td>Nuclei</td>
<td>&lt; 1 pm</td>
</tr>
</tbody>
</table>

Radios and Antennas

Radio Transmission

- We need to attach a message to a carrier wave, transmit it, and then decode it.
- Carrier wave – high frequency
  - AM 500-1600 kHz
  - FM 88-110 MHz
- Audio signal
  - 20Hz – 20 kHz

Two Waves

- Signal Wave
- Carrier Wave

Amplitude Modulation (AM)

- modulate the amplitude of the carrier wave by the signal wave.

Phase Modulation (PM)

- modulate the phase of the carrier wave by the signal wave.
Frequency Modulation (FM)
• modulate the frequency of the carrier wave by the signal wave. Much like PM

Transmitting Antennas
• Connect an oscillator to bare wires.

Transmitting Antennas
• Each electron becomes a source of dipole radiation.

Transmitting Antennas
• By integrating over each little wire segment, we can find the radiation fields.

Antenna Patterns
• By making more complicated arrangements of antennas, we can make beams that radiate more power in specific directions.
• The physics of multi-element antennas is similar to multiple slit diffraction in optics.

Receiving Antennas
• Receiving antennas are much like transmitting antennas.
• The electric field in a radio wave causes electrons in the antenna to oscillate at the frequency of the carrier wave.
• The antenna then becomes a high-frequency AC source.
Receiving Antennas

- We then connect an antenna to a series LRC circuit so we can tune the circuit.

![](antenna.png)

Receiving Antennas

- We adjust the variable capacitor so the circuit oscillates at the carrier frequency.

![](antenna.png)

Receiving Antennas

- The voltage across the resistor can then be amplified and the signal separated from the carrier.

![](antenna.png)

Class 41

Today we will:

- learn how digital information is transmitted on electromagnetic waves
- learn the meaning of polarization
- learn about polarized light and its applications

Transmitting Information on EM Waves

Transmitting Digital Data

- To transmit digital data, all we need to do is turn the carrier on and off, or better, transmit the wave with two different amplitudes.

![](signal.png)
Transmitting Digital Data

• But you can’t change the two amplitudes much faster than once a wavelength.

Baud Rate

• Baud rate is number of bits (binary integers) that are transferred per second.
• The baud rate on any electromagnetic wave is limited to approximately the frequency of the wave.
• Waves with short wavelength or high frequency can transfer data at higher rates.

Bandwidth

• In common terminology, bandwidth often means the same thing as baud rate.
• Technically, bandwidth means the range of frequencies that are available for transmissions. It is used in two senses.

Bandwidth - 1

• The range of frequency required for a given signal to be clearly transmitted and received. – For example – how close in frequency two signals can be together and the signals not be confused.
• FM signals require greater bandwidth than AM signals because the frequency is modulated.

Bandwidth - 2

• The range of frequencies allocated for transmission, so that several transmissions can be broadcast simultaneously.
• The broader the bandwidth in this sense, the more data can be transferred.

Polarization
The Fields of a Simple Antenna
• Take a simple antenna with electrons oscillating along the length of the antenna.
  • Threads arriving at $P$ came from a charge accelerating to the right.

The Fields of a Simple Antenna
• The direction of the electric field is $\vec{R} \times (\vec{R} \times \hat{a})$

The Fields of a Simple Antenna
• The direction of the magnetic field is $\vec{E} \times \vec{B}$

The Fields of a Simple Antenna
• Now take another point, a little farther out, so threads arriving here were emitted when acceleration was to the left.

The Fields of a Simple Antenna
• Finally, take a third point...

The Fields of a Simple Antenna
• Note that the electric field oscillates back and forth at the same frequency as the frequency of the oscillations in the antenna.
  • The electric field changes in magnitude, but it is always parallel to the antenna.
  • The magnetic field is always into the screen and out of it.
Polarization
• We say that the beam is polarized in the directions of the electric field.
• In this case, the wave is horizontally polarized.

Many Sources
• If there are many oscillators, they may oscillate in the same direction, as different electrons in an antenna.
• They may oscillate in random directions, as in a light bulb, or the sun.

Unpolarized Light
• We say light from the sun is unpolarized.
  - We know, however, that the electric field of light from the sun must lie in a plane perpendicular to the direction of the ray's travel.

Unpolarized Light
• In this case, the plane of polarization is the plane of the screen.

Polarization by Reflection and Scattering
• An oscillating electron is like a little dipole antenna.
• It radiates most strongly in the plane perpendicular to its line of motion.

Polarization by Reflection
• Let’s assume light from the sun is polarized horizontally.
• The E field of the light causes electrons on the surface of a lake to oscillate horizontally.
Polarization by Reflection

- The electrons in the water radiate in the plane of the screen – some radiate toward the observer.

- If the surface is smooth, the incident angle equals the reflected angle.

Polarization by Reflection

- Now let’s assume light from the sun is polarized the other way.
- The $E$ field of the light causes electrons to oscillate in the direction of the red arrows.

- Therefore light reaching the observer is primarily polarized in the horizontal direction.

Polarization by Scattering

- The same effect happens when light scatters, except that the oscillating electrons are spread throughout the atmosphere.
Polarization by Scattering
• When the angle between the incident ray and the scattered ray is 90º, the polarization is largest.

Determining the Polarization Direction
• An easy way to determine the polarization direction: It lies along the line that intersects the polarization plane of the incident ray and the polarization plane of the reflected or scattered ray.

How Do You Tell If Light Is Polarized?
• A polarizing filter allows only the part of the light that is polarized along its axis to pass.
• Therefore a polarizing filter also polarizes light.

Polarization by Birefringence
• Some crystals, such as calcite, refract light differently depending on its polarization direction.
• These are called “birefringent.”

How Do You Tell If Light Is Polarized?
• If you rotate the filter and the intensity of the light changes, the light is at least partially polarized.
• Polaroid sunglasses are polarizing filters.

Polarization with Polarizers
• When unpolarized light passes through a polarizing filter, half the intensity is lost.
• Once light is polarized, we keep track of the electric field strength.
Polarization with Polarizers

• We break down the electric field vector into components parallel and perpendicular to the polarizer axis.

\[ \frac{I_0}{2} = E_1 \cos \theta \rightarrow \theta \rightarrow E_1 \cos \theta \]

Polarization with Polarizers

• The intensity is proportional to the square of the electric field.

\[ \frac{I_1}{I_0} = \frac{E_1^2 \cos^2 \theta}{E_1^2} = \cos^2 \theta \]

\[ \frac{I_2}{I_0} = \frac{1}{2} \cos^2 \theta \]

Malus’s Law

• For transmission of polarized light through polarizing filters.

\[ \frac{I_{\text{out}}}{I_{\text{in}}} = \cos^2 \theta \]