Class 34

Today we will:
• learn about inductors and inductance
• learn how to add inductors in series and parallel
• learn how inductors store energy
• learn how magnetic fields store energy
• learn about simple LR circuits

Inductors
An inductor is a coil of wire placed in a circuit. Inductors are usually solenoidal or toroidal windings.

An Inductor in a DC Circuit
In a DC circuit an inductor behaves essentially like a long piece of wire – except the solenoid produces a magnetic field and the wire has a resistance.

An Inductor in an AC Circuit
In an AC circuit, the current is continually changing. Hence the magnetic field in the inductor is continually changing.

An Inductor in a DC Circuit
If we bring a magnet near the inductor, an induced EMF is produced that alters the voltages and current in the circuit.

An Inductor in an AC Circuit
If the magnetic field in the inductor changes, an EMF is produced. Since the inductor’s own field causes an induced EMF, this is called “self inductance.”
The Inductance of a Solenoid

We can easily calculate the EMF of a solenoidal inductor.

\[
B = \mu_0 ni
\]

\[
\Phi_B = BA = \mu_0 niA
\]

\[
V = -N \frac{d\Phi_B}{dt} = -N\mu_0 nA \frac{di}{dt}
\]

\[
V = -\mu_0 n^2 \ell A \frac{di}{dt}
\]

Units of Inductance

\[
V = -L \frac{di}{dt}
\]

Inductance is in units of henrys (H).

\[
1 H = 1 \frac{V}{A/s} = 1 \Omega s
\]

Inductance in Series and Parallel

In series:

\[
V = V_1 + V_2
\]

\[
-L \frac{di}{dt} = -L_1 \frac{di_1}{dt} - L_2 \frac{di_2}{dt}
\]

\[
\Rightarrow L = L_1 + L_2
\]

In parallel:

\[
i = i_1 + i_2
\]

\[
\Rightarrow \frac{di}{dt} = \frac{di_1}{dt} + \frac{di_2}{dt}
\]

\[
\frac{V}{L} = \frac{V}{L_1} + \frac{V}{L_2}
\]

\[
\Rightarrow \frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}
\]

Voltages of Circuit Elements

\[
V = Ri = R \frac{dq}{dt}
\]

\[
V = \frac{1}{C} q
\]

\[
V = -L \frac{di}{dt} = -L \frac{d^2q}{dt^2}
\]

Note that \( L \) is similar to \( R \) in these equations.

Energy in an Inductor

Take a circuit with only an EMF and an inductor.

\[
\varepsilon = -V_L = L \frac{di}{dt}
\]
Energy in an Inductor

\[ \varepsilon = L \frac{di}{dt} \]

\[ P = i \varepsilon = iL \frac{di}{dt} \]

\[ = \frac{d}{dt} \frac{1}{2} Li^2. \]

Energy in a Capacitor

Recall that the energy stored in a capacitor is:

\[ U = \frac{1}{2} CV^2 \]

Energy Density in a Magnetic Field

Since \( B = \mu_0 ni \)

\[ U = \frac{1}{2\mu_0} (\mu_0 ni)^2 \text{ vol} \]

\[ u = \frac{U}{\text{ vol}} = \frac{1}{2\mu_0} B^2 \]

Energy Density in an Electric Field

Recall that in an electric field:

\[ u = \frac{U}{\text{ vol}} = \frac{1}{2} \varepsilon_0 E^2 \]
LR Circuits
• An LR circuit consists of an inductor, a resistor, and possibly a battery.
• Inductors oppose change in circuits.

\[ V \rightarrow L \rightarrow R \]

LR Circuits
• In steady state, inductors act like wires, but they significantly affect circuits when switches are opened and closed.

\[ V \rightarrow L \rightarrow R \]

LR Circuits
• When the switch is closed in this circuit, the inductor opposes current flow. Initially, it is able to keep any current from flowing.

\[ V \rightarrow L \rightarrow R \]

LR Circuits
• Current then increases until it reaches a maximum value, \( i = \frac{V}{R} \).

\[ V \rightarrow L \rightarrow R \]

LR Circuits
• Current in an LR circuit is very similar to charge in an RC circuit.

\[ V \rightarrow L \rightarrow R \]

\[ \frac{V}{R} \left(1 - \frac{1}{e^{\frac{t}{\tau}}}\right) \]
LR Circuits
• Think of the total current being a combination of the induced battery’s current and the induced current.
• The current from the battery is $I = \frac{V}{R}$, and it is constant.

LR Circuits
• The induced current is initially $I = -\frac{V}{R}$.
• The induced current drops to zero as time increases.

LR Circuits
• If the inductor is large, it is more effective at making the induced current, so it takes a longer time for the induced current to decrease.
• If the resistor is large, it is more effective at decreasing the induced current, so it takes a shorter time for the current to decrease.

LR Circuits
• We’ll show next time that the inductive time constant is:

\[ \tau = \frac{L}{R} \]

Class 35
Today we will:
• learn more about LR circuits
• learn the LR time constant
• learn about LC circuits and oscillation
• learn about phase angles

LR Circuits
• An LR circuit consists of an inductor, a resistor, and possibly a battery.
• Inductors oppose change in circuits.
The Definition of Inductance

We define inductance by the equation:

\[ V = -L \frac{di}{dt} \]

Kirchoff’s Loop Law

• The voltage around the loop at any given time must be zero.
• It’s important to be careful of signs.

Kirchoff’s Loop Law

• As the current increases, the inductor opposes the increase. It produces an EMF in opposition to the battery.
Kirchoff's Loop Law

\[ V - L \frac{di}{dt} - iR = 0 \]

\[ \frac{di}{dt} > 0 \]

\[ \Rightarrow V - L \frac{di}{dt} - iR = 0 \]

Another LR Circuit

• We can also make an LR circuit in which the current decreases.
• At \( t=0 \), the switch is moved from 1 to 2.

\[ i(t) = \frac{V}{R} \left( 1 - e^{-\frac{t}{\tau}} \right) \]

\[ \Rightarrow V - L \frac{dv}{dt} e^{-\frac{t}{\tau}} + V e^{-\frac{t}{\tau}} = 0 \]

\[ \Rightarrow \tau = \frac{L}{R} \]

Kirchoff's Loop Law

• We apply Kirchoff's Laws again.
• This time the induced EMF pushes charge forward.

\[ V - L \frac{di}{dt} - iR = 0 \]
Kirchoff’s Loop Law

\[ L \frac{di}{dt} - iR = 0 \]

\[ \frac{di}{dt} < 0 \]

\[ -L \frac{di}{dt} - iR = 0 \]

\[ \begin{array}{c}
V \\
R \\
L \\
\end{array} \]

\[ i(t) = \frac{V}{R} e^{-\frac{t}{\tau}} \]

\[ + \frac{V}{R} e^{\frac{t}{\tau}} - R \frac{V}{R} e^{-\frac{t}{\tau}} = 0 \]

\[ \Rightarrow \tau = \frac{L}{R} \]

\[ \begin{array}{c}
V \\
R \\
L \\
\end{array} \]

LC Circuits

- An LC circuit consists of an inductor and a capacitor.

\[ \begin{array}{c}
C \\
L \\
\end{array} \]

LC Circuits

- Initially the capacitor is charged.
- No current flows as the inductor initially prevents it.
- Energy is in the electric field of the capacitor.

\[ \begin{array}{c}
C \\
L \\
\end{array} \]
LC Circuits

• The charge reduces.
• Current increases.
• Energy is shared by the capacitor and the inductor.

LC Circuits

• The charge goes to zero.
• Current increases to a maximum value.
• Energy is in the magnetic field of the inductor.

LC Circuits

• The inductor keeps current flowing.
• The capacitor begins charging the opposite way.
• The capacitor and inductor share the energy.

LC Circuits

• The capacitor becomes fully charged.
• Current stops.
• All the energy is in the capacitor.

LC Circuits

• The capacitor begins to discharge.
• Current flows counterclockwise.
• The capacitor and inductor share the energy.

LC Circuits

• Without energy loss, current would continue to oscillate forever.
Kirchoff’s Loop Law

• First, we’ll be really sloppy about signs.

\[ +L - C_i \]

Kirchoff’s Loop Law

• First, we’ll be really sloppy about signs.
• Know this!

\[ +L - C_i \]

\[ \text{But that’s not oscillatory!} \]

Kirchoff’s Loop Law

\[ V_c = V_L \]
\[ q = \frac{L}{C} \frac{di}{dt} = \frac{L}{C} \frac{d^2q}{dt^2} \]
\[ \frac{d^2q}{dt^2} = \frac{1}{LC} q(t) \]
\[ q(t) = Ae^{-\beta t} + Be^{\beta t} \]

If the charge is maximum \( Q=CV_0 \) at \( t=0 \), then we have:

\[ q(t) = CV_0 \cos \omega t \]
Oscillating Frequency

\[ \frac{d^2 q}{dt^2} = -\frac{1}{LC} q(t) \]
\[ q(t) = CV_0 \cos \omega t \]
\[ -\omega^2 CV_0 \cos \omega t = -\frac{1}{LC} CV_0 \cos \omega t \]
\[ \Rightarrow \omega = \frac{1}{\sqrt{LC}} \]

Being Careful about Signs

• Whenever we have AC circuits, we have to be really careful about signs. Signs tell us directions, and directions can be confusing.

Resistors

• Choose a direction for positive current.

  \[ +i \quad \begin{array}{c}
  \text{Resistors}
  \end{array} \]

  \[ +i \quad \begin{array}{c}
  \text{Resistors}
  \end{array} \]

  \[ V = iR > 0 \]

  \[ V = iR < 0 \]

  \[ \text{positive voltage} \]
Resistors

- Choose a direction for positive current.
- We use $V = iR$ to give the voltage across a resistor.
- We call $V$ positive when the voltage pushes current in the negative direction.
- We call $V$ negative when the voltage pushes current in the positive direction.

$V_R = iR < 0$

negative voltage

Resistors

- The voltage and current are in phase.
- The phase angle is 0°.

Capacitors

- Choose a direction for positive current.

Capacitors

- Choose a direction for positive current.
- If the left plate is positive, $q > 0$ and $V > 0$.
- If the left plate is negative, $q < 0$ and $V < 0$.

Capacitors

- We plot charge and voltage as a function of time.

Capacitors

- If current is flowing right, $dq/dt > 0$.
- If current is flowing left, $dq/dt < 0$. 
Capacitors

- If current is flowing right, $dq/dt > 0$.
- If current is flowing left, $dq/dt < 0$.

\[ i = \pm dq/dt \]

$q$ becoming more positive

- If current is flowing right, $dq/dt > 0$.
- If current is flowing left, $dq/dt < 0$.

$q$ becoming more positive

When $q$ (or $V$) increases, the current is positive.
When $q$ (or $V$) decreases, the current is negative.

The voltage lags the current.
The phase angle is $-90^\circ$.

Inductors

- Choose a direction for positive current.

Choose a direction for positive current.
We need to consider current in both directions.
In each direction, we need to consider current increasing and decreasing.
Inductors

• In each direction, we need to consider current increasing and decreasing.

\[
\begin{align*}
\text{Induced } i &< 0 \\
\frac{di}{dt} &> 0 \\
\text{increasing} &
\end{align*}
\]

\[
\begin{align*}
\text{Induced } i &> 0 \\
\frac{di}{dt} &< 0 \\
\text{decreasing} &
\end{align*}
\]

- The induced EMF opposes the current.
- The voltage positive because it is pushing current in the negative direction.

\[
V_L = L \frac{di}{dt} > 0
\]

positive voltage

Inductors

• In each direction, we need to consider current increasing and decreasing.

\[
\begin{align*}
\text{Induced } i &< 0 \\
\frac{di}{dt} &> 0 \\
\text{increasing} &
\end{align*}
\]

\[
\begin{align*}
\text{Induced } i &> 0 \\
\frac{di}{dt} &< 0 \\
\text{decreasing} &
\end{align*}
\]

- The induced EMF aids the current.
- The voltage negative because it is pushing current in the positive direction.

\[
V_L = L \frac{di}{dt} < 0
\]

negative voltage
Inductors

- In each direction, we need to consider current increasing and decreasing.

\[
i(t) = \begin{cases} 
  i > 0 & \text{increasing} \\
  i < 0 & \text{decreasing} 
\end{cases}
\]

- The induced EMF opposes the current.
- The voltage is negative because it is pushing current in the positive direction.

\[
i(t) \quad \frac{di}{dt} < 0 \quad \Rightarrow \quad V_L = L \frac{di}{dt} < 0
\]

- The voltage is negative when \(\frac{di}{dt}\) is negative.
- The voltage is positive when \(\frac{di}{dt}\) is positive.

Inductors

- The voltage leads the current.
- The phase angle is +90°.

Voltage and Current in an LC Circuit
Today we will:
• learn about phasors
• define capacitive and inductive reactance
• learn about impedance
• apply Kirchoff’s laws to AC circuits

Voltage and Current in AC Circuits
ELI the ICE MAN

In an inductor (L) EMF leads I

In a capacitor (C) I leads EMF

Representing an AC Current
• If a current varies sinusoidally, we usually represent it graphically.
• But we can also represent it with an arrow that goes up and down in time.
Representing an AC Current

- An arrow that goes up and down sinusoidally is just the $y$ component of a vector that rotates at a constant angular frequency.
- The length of the arrow is $i_0$, and the angle of the vector is $\theta = \omega t$.

Phasors

- A phasor is a vector that represents a sinusoidal function. Its magnitude is the amplitude of the sine wave (it’s maximum value) and its angle is the phase angle (the argument of the sine function).

Phasors

- Phasors are useful because we can add two sine waves with the same frequency by adding their phasors.

$$i(t) = i_0 \sin \theta$$

Phasors

- Doesn’t this just make things harder???

- No – because phasor diagrams are a lot easier than algebra and trig identities.

- Let’s look at some examples...

Phasors

- First, we wish to find the maximum voltage across a resistor in terms of the maximum current.
- Note these correspond to the length of the phasors.
- All we need for this is Ohm’s Law: $V_{max} = i_0 R$
Resistors and Phasors

• Now we can make a voltage phasor and a current phasor for the resistor.
• The voltage is in phase with the current, so the phasors point in the same way.

\[
\text{if } R = 2\, \Omega \quad \begin{array}{c}
\text{i} \\
\text{V} \\
R
\end{array}
\]

Inductors and Phasors

• As before, we wish to find the maximum voltage across an inductor in terms of the maximum current.

\[
\text{Let } i(t) = i_0 \sin(\omega t) \\
\text{Then } V_{\text{i}}(t) = L \frac{di}{dt} = L \omega i_0 \cos(\omega t) \\
\text{So } V_{\text{i,0}} = L \omega i_0
\]

Inductive Reactance

• This makes sense, because an inductor offers more opposition to the current if the frequency is large and if the inductance is large.

\[
V_{\text{i,0}} = i_0 X_L \\
X_L = \omega L
\]

Inductive Reactance

• Comparing this result to Ohm’s Law, we see that \(\omega L\) has a role that is similar to resistance – it relates voltage to current.
• We define this to be the “inductive reactance” and give it a symbol \(X_L\).

\[
V_{\text{i,0}} = i_0 X_L \\
X_L = \omega L
\]

Inductors and Phasors

• We again want a voltage phasor and a current phasor.
• The voltage leads the current by 90°, so the phasors are:

\[
\text{if } X_L = 2\, \Omega \quad \begin{array}{c}
\text{V_L} \\
\text{i}
\end{array}
\]
Inductors and Phasors

Capacitors and Phasors

• We find the maximum voltage across a capacitor in terms of the maximum current.

\[ i(t) = i_0 \sin(\omega t) \]

Then \[ V_C(t) = \frac{q(t)}{C} = \frac{1}{C} \int i(t) \, dt = -\frac{1}{\omega C} i_0 \cos(\omega t) \]

as \[ i = \frac{dq}{dt} \]

So \[ V_{c0} = \frac{1}{\omega C} i_0 \]

Capacitive Reactance

• Comparing this result to Ohm’s Law, we see that \( i \omega C \) is similar to resistance.

• We define this to be the “capacitive reactance” and give it a symbol \( X_C \).

\[ V_{c0} = i_0 X_C \]

\[ X_C = \frac{1}{\omega C} \]

Capacitors and Phasors

• We again want a voltage phasor and a current phasor.

• The voltage lags the current by 90°, so the phasors are:

\[ V_C = i_0 X_C \]

\[ X_C = \frac{1}{\omega C} \]

Capacitive Reactance

• This makes sense because a capacitor offers more resistance to current flow if it builds up a big charge. If the frequency is large or the capacitance is large, it is hard to build up much charge.
Circuit Rules for AC Circuits

If two circuit elements are in parallel:

1) their voltage phasors are the same.

\[ V_L = V_C \]

Circuit Rules for AC Circuits

If two circuit elements are in series:

1) their current phasors are the same.

\[ i_L = i_C \]

2) their voltage phasors add to give the total voltage.

\[ V_L = V_C \]
An Example

We know $\varepsilon_0$, $\omega$, the resistances, the capacitance, and the inductance.

\[ \varepsilon \]

\[ R_1 \]

\[ C \]

\[ \omega \]

\[ L \]

\[ \varepsilon_1 \]

\[ R_2 \]

An Example

Label the currents.

\[ \varepsilon \]

\[ i_1 \]

\[ R_1 \]

\[ i_2 \]

\[ R_2 \]

\[ L \]

\[ C \]

\[ i_3 \]

An Example

We don’t know the currents, but we know the relationships between current and voltage.

\[ \varepsilon \]

\[ i_1 \]

\[ R_1 \]

\[ i_2 \]

\[ R_2 \]

\[ L \]

\[ C \]

\[ i_3 \]

An Example

\[ \varepsilon \]

\[ i_1 \]

\[ R_1 \]

\[ i_2 \]

\[ R_2 \]

\[ L \]

\[ C \]

\[ i_3 \]

\[ i_{20} = i_1 R_1 \]

An Example

\[ \varepsilon \]

\[ i_1 \]

\[ R_1 \]

\[ i_2 \]

\[ R_2 \]

\[ L \]

\[ C \]

\[ i_3 \]

\[ V_{x20} = i_2 R_2 \]

An Example

\[ \varepsilon \]

\[ i_1 \]

\[ R_1 \]

\[ i_2 \]

\[ R_2 \]

\[ L \]

\[ C \]

\[ i_3 \]

\[ V_{L0} = i_{20} X_L \]
An Example

We can combine circuit elements and calculate the total impedance of these elements.

We can combine all the circuit elements into a single impedance.

We’ll start with the inductor.

Draw the current in an arbitrary direction – along the x-axis will do – with an arbitrary length.
An Example
We know the direction of the voltage phasor from “ELI.”

\[ V_{L0} = i_{20} X_L = i_{20} \omega L \]

An Example
We know the magnitude of the voltage phasor is \( \omega L \) times \( i_{20} \).

An Example
Now we add the resistor.

\[ V_{R0} = i_{20} R_2 \]

An Example
The voltage phasor is in the same direction as the current. It’s magnitude comes from:

\[ V_{R0} = i_{20} R_2 \]

An Example
We add the voltages of the two series elements.

\[ V_{R0} = i_{20} R_2 \]

An Example
Let’s look at the math.
An Example
The LR voltage phasor has two components:
\[ \vec{V}_{LR} = V_{RL} \hat{x} + V_{LR} \hat{y} \]
\[ V_{RL} = i_0 R_L, \quad V_{LR} = i_0 X_L \]

An Example
The length of the LR voltage phasor is:
\[ \vec{V}_{LR} = V_{RL} \hat{x} + V_{LR} \hat{y} \]
\[ V_{RL} = i_0 R_L, \quad V_{LR} = i_0 X_L \]
\[ V_{LR0} = \sqrt{V_{RL}^2 + V_{LR}^2} = i_0 \sqrt{R_L^2 + X_L^2} \]

Impedance
Impedance is the combined “effective resistance” of circuit elements. It is given the symbol \( Z \).
\[ V_{LR0} = i_0 Z_{LR} \]

An Example
We can define a total impedance for L and R:
\[ \vec{V}_{LR} = V_{RL} \hat{x} + V_{LR} \hat{y} \]
\[ V_{RL} = i_0 R_L, \quad V_{LR} = i_0 X_L \]
\[ V_{LR0} = \sqrt{V_{RL}^2 + V_{LR}^2} = i_0 \sqrt{R_L^2 + X_L^2} = i_0 Z_{LR} \]

An Example
We can calculate the phase angle of the LR phasor:
\[ \phi_{LR} = \tan^{-1} \frac{V_{LR}}{V_{RL}} = \tan^{-1} \frac{X_L}{R_L} \]
Impedance
Whenever we know a voltage and a current we can find an impedance. Because of differences in phases, impedances are not added as easily as in DC circuits.

An Example
Now let’s continue the problem, without filling in all the mathematical details.

An Example
We replace the inductor and resistor with a single impedance:

\[
\begin{align*}
\mathcal{E} & \hspace{1cm} i_1 \hspace{1cm} R_1 \\
i_2 & \hspace{1cm} Z_{12} \\
C & \hspace{1cm} i_3
\end{align*}
\]

The \( LR \) voltage phasor is the same as the \( C \) voltage phasor, since \( LR \) and \( C \) are in parallel.

An Example
We can find the direction and magnitude of \( i_3 \).

\[
\begin{align*}
\mathcal{E} & \hspace{1cm} i_1 \hspace{1cm} R_1 \\
i_2 & \hspace{1cm} Z_{12} \\
C & \hspace{1cm} i_3
\end{align*}
\]

The direction of \( i_3 \) is given by “ICE.”
An Example

The magnitude of $i_3$ is given by:

$$V_{Cu} = V_{LRC} \Rightarrow i_3 = i_3 Z_{r}$$

The total current through LRC is $i_{LRC} = I_2 + i_1$.

By Kirchoff’s junction rule, we have $i_1 = i_{LRC}$.

We can replace LRC with a single impedance.

Now we can find the voltage across the resistor with Ohm’s Law.

Finally, we can add the two voltages to get the EMF.
Class 37

Today we will:
• study the series LRC circuit in detail.
• learn the resonance condition
• learn what happens at resonance
• calculate power in AC circuits

The Series LRC Circuit
• Assume we know the maximum voltage of the AC power source, $\epsilon_0$.
• We wish to find the current and the voltages across each circuit element.

$$\begin{align*}
\epsilon + V_R + V_L + V_C &= 0 \\
V_{Rs} &= i_R R \\
V_{Cs} &= i_L X_L = i_L \sigma L \\
V_{Cs} &= i_C X_C = \frac{i_C}{\omega C} \\
\epsilon_a &= i_a Z
\end{align*}$$
• Since $i$ is a common factor in all the voltages, we can divide it out and simplify the math a little.

• Let’s redo this diagram step by step. Be sure you know this well!

• Start with the current direction. It’s easiest just to put this on the +x axis.

• Put $X_L$ along the +y axis.

• Put $X_C$ along the −y axis.

• Put $R$ along the +x axis.
The Series LRC Circuit

- Now we add all the impedances as vectors.
- First, add $X_L$ and $X_C$.

\[ i \]
\[ R \]
\[ L \]
\[ C \]
\[ X_L \]
\[ X_C \]

The Series LRC Circuit

- Then add $R$ to get $Z$.

\[ i \]
\[ R \]
\[ L \]
\[ C \]
\[ X_L \]
\[ X_C \]

The Series LRC Circuit

- We know $\varepsilon = \varepsilon_0$ and $\varepsilon_i = i_0 Z$.

\[ i \]
\[ R \]
\[ L \]
\[ C \]
\[ X_L \]
\[ X_C \]

The Series LRC Circuit

- We can find $Z$ and $i_0$:
  \[ Z = \sqrt{(X_L - X_C)^2 + R^2} \]
  \[ i_0 = \frac{\varepsilon_0}{Z} \]

\[ i \]
\[ R \]
\[ L \]
\[ C \]
\[ X_L \]
\[ X_C \]

Varying the Frequency

- As we vary the frequency, the phasor diagram changes.
- At low frequency, the capacitive reactance is large.
- At high frequency, the inductive reactance is large.
Varying the Frequency

• In this animation, we keep the maximum EMF constant and increase the frequency of the AC power supply.

Resonance

• Resonance is where the inductive and capacitive reactances are equal.

\[ X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C} \]

\[ \omega = \frac{1}{\sqrt{LC}} \]

• The resonant frequency is:

\[ X_L = X_C \Rightarrow \omega L = \frac{1}{\omega C} \]

\[ \omega = \frac{1}{\sqrt{LC}} \]

• The impedance is minimum and the current is maximum.

\[ Z = \sqrt{(X_L - X_C)^2 + R^2} = R \]

Power in AC Circuits

• At any given instant, the power dissipated by a resistor is

\[ P = i(t)V(t) \]

\[ = Ri^2 = R\varepsilon_0^2 \sin^2(\alpha) \]

• Over a full cycle of period \( T \), the average power dissipated in the resistor is:

\[ \bar{P} = \frac{1}{T} \int_0^T P(t)dt = \frac{R\varepsilon_0^2}{T} \int_0^T \sin^2(\alpha)dt = \frac{R\varepsilon_0^2 T}{T} \]

\[ \bar{P} = \frac{1}{2} i_0 V_0 = \frac{1}{2} i_0 V_0 \]

Power in AC Circuits

• The power provided by a voltage source is:

\[ P(t) = i(t)\varepsilon(t) \]

\[ = i_0 \varepsilon \sin(\alpha) \sin(\alpha + \phi) \]

\[ = i_0 \varepsilon \sin(\alpha) \left[ \sin(\alpha) \cos \phi + \cos(\alpha) \sin \phi \right] \]

\[ \bar{P} = \frac{1}{2} i_0 \varepsilon \cos \phi \]
rms Currents and Voltages

• We want to define an “effective AC current.”

• The effective current is less than the maximum current.

• The average current is zero.

So, we take the square root of the average value of the current squared.

This is called the rms or “root mean square” current.

\[ i_{\text{rms}} = \sqrt{\frac{1}{T} \int_0^T i(t)^2 \, dt} = \frac{1}{\sqrt{2}} i_0 \]

• Similarly:

\[ V_{\text{rms}} = \frac{1}{\sqrt{2}} V_0 \]

rms Currents and Voltages

• We want to define an “effective AC current.”

• The effective current is less than the maximum current.

• The average current is zero.

• Absolute values are awkward mathematically.

 rms Currents and Voltages

• Normally, when we specify AC voltages or currents, we mean the rms values.

• Standard US household voltage is 110-120V rms.
Power in Terms of rms Quantities

• When we rewrite our power equations in terms of rms currents and voltages, they become similar to DC formulas.

Resistor
\[ P = \frac{1}{2} i_{\text{rms}} V_{\text{rms}} = i_{\text{rms}}^2 R = \frac{V_{\text{rms}}^2}{R} \]

Power supply:
\[ P = \frac{1}{2} i_{\text{rms}} e_{\text{rms}} \cos \phi = i_{\text{rms}} e_{\text{rms}} \cos \phi \]