PHSCS 751 **HWK # 8** **Due 11/5/09**

Question One.

Argue that the α^k and β operators are Hermitian and then show it explicitly by using their matrix representation. What can you say about the hermiticity of the γ^μ operators?

Question Two.

Find the relativistically covariant generalization of Strange equation (4.19a) by separating the symmetric and antisymmetric parts of a product of two γ operators

$$a_\mu \gamma^\mu \;\; b_\nu \gamma^\nu = a_\mu b^\mu - i a_\mu b_\nu \sigma^{\mu\nu},$$

with $\sigma^{\mu\nu}$ defined as in Eq (4.83) but without the action units.

Question Three.

Express the Gordon decomposition in relativistically covariant (Greek index) notation. You may either "fix" Strange equations (4.107) and (4.108) or rederive it from the relativistically covariant Dirac equation (and its conjugate) and your result in Question Two above.

Question Four.

Find the eigenvalues and eigenvectors of the velocity operator given in Eq. (4.113a).

Question Five.

Check the gauge invariance of the Dirac equation without making the time-independence assumption that Strange makes in section 4.9. In other words take $\theta(r, t)$ and use the time-dependent Dirac equation. You can check your derivation and solution against the similar discussion of the gauge invariance of the Schrödinger equation in Merzbacher section 4.6 or Griffiths' problem 4.61.