ACCELERATION DUE TO GRAVITY

Skills you will learn or practice:
- Calculate velocity and acceleration from experimental measurements of x vs t (spark positions)
- Find average velocities from two positions and times, and average accelerations from two velocities and times
- Sketch v(t) from the slope of x(t) and a(t) from the slope of v(t)
- Extract physical constants from polynomial and linear fits to data
- Use the kinematic equations to relate time, position, velocity when acceleration is constant
- Percent error

Conceptual understanding:
- Linear vs polynomial equations and graphs
- How the equipment works as explained in the lab manual
- Other conceptual questions the lab manual suggests you consider.

OVERVIEW
Before each lab you should carefully read this lab and write an overview of what you will do in the lab. Describe the goals of the lab and how they will be accomplished. It should be no more than 1/3 of a page. Your TA will look for this at the beginning of lab as evidence of your preparation.

PHYSICAL PRINCIPLES
Before coming to the lab you should review from your textbook and class notes the idea of average and instantaneous velocity and acceleration, and the kinematic equations and how they relate to each other.

INTRODUCTION
You will study the motion of an object moving in a straight line as it falls. This involves measurement of the location of the object (the position x on some reference line) at many times. You can then approximate its instantaneous velocity at some time by calculating the change that occurs in x between two known, closely spaced times. As discussed in your textbook, instantaneous velocity and instantaneous acceleration can be expressed by the equations

\[ v = \frac{\Delta x}{\Delta t} \quad \text{and} \quad a = \frac{\Delta v}{\Delta t}, \]

where the symbols \( \Delta x, \Delta v, \) and \( \Delta t \) mean small intervals or changes in \( x, v, \) and \( t, \) respectively.

In this experiment you will record the position of a falling object by a spark on a paper tape. The figure below shows a tape with dots that represents the position of an object moving at constant speed at equal time intervals (perhaps a toy train on a track with one dot every second). The speed is constant so the dots are evenly spaced. The graph of the data x(t) is also sketched. The slope of x(t) is constant, so v(t) is a constant, as sketched to the right. What is the acceleration?
If the object is accelerating, the dots won’t be evenly spaced, and the velocity won’t be constant. You can measure an instantaneous velocity by measuring the distance \( \Delta x \) an object travels in a short period of time \( \Delta t \), and you can measure an instantaneous acceleration by determining two velocities that are separated by a known small time difference. The velocity and the acceleration determined by this process are actually average values over the small interval, but are very nearly the instantaneous value at the center of the time interval. Such measurements can tell you how the velocity and acceleration vary in time.

When you plot curves of \( x(t) \) and \( v(t) \), the slope of the \( x(t) \) curve at each \( t \) value represents the velocity at that time; the slope of the \( v(t) \) curve at each time represents the acceleration of the object at that time. While we will use them to describe motion, these ideas of *rates* and *rate changes* are important in many fields, including economics, biology and sociology.

You will study the motion of an object falling freely at constant acceleration \( g \) under the influence of gravity. Your textbook mathematically develops equations for \( x(t) \) and \( v(t) \):

1. \( x = x_0 + v_0 t + \frac{1}{2} a t^2 \)
2. \( v = v_0 + a t \)
3. \( v^2 = v_0^2 + 2a(x-x_0) \)

Note that these “kinematic equations” for constant acceleration are *polynomials*. Equation 1 is a second order polynomial of the form, \( y=A_0+A_1x+A_2x^2 \), and equation 2 is a first order polynomial or linear equation of the form, \( y=mx+b \).

**PROCEDURE**

To study the motion of a free falling object, you will first verify that the acceleration due to gravity is in fact constant (when air friction is negligible). You will then compare the form of your plots of \( x(t) \) and \( v(t) \) with the equations given. From this comparison you should be able to determine a precise value for \( g \), the acceleration due to gravity.

To experimentally check whether the acceleration is truly constant, the velocity at many known times must be measured. You can make such measurements by using an electronic spark timer, a device that produces repetitive, high-voltage sparks at regular intervals. You will use an apparatus that allows a metal cylinder to fall freely between two straight vertical wires, as illustrated in Fig. 1a. At known time intervals a spark jumps from one wire to the ridge of the cylinder, and through the metal, to the other wire. The spark leaves a spot, that you can measure, on heat-sensitive paper. Because you know the time interval of the spark (precisely 60 times/sec), this technique specifies the location of the metal cylinder at a large number of known times.
A. The Experiment

Before you perform the experiment, you must make specific adjustments in order to vertically align the apparatus. The apparatus should be aligned but you may want to verify this. Here are a few hints and cautions on adjusting the equipment and performing the experiment.

1. The cylinder must fall vertically between the wires and stay close to them at all times. Hang the plumb bob from the electromagnet and adjust the tilting screws at the base of the apparatus until the plumb bob hangs precisely between the wires.

2. After adjusting the tilting screws, hang the metal cylinder from the electromagnet and press the release button at the end of the white wire. Make sure it falls straight and lands on the clay disk on the floor. Practice the procedure once or twice.

3. Attach the heat sensitive paper, with masking tape, along the inside of one of the wires.

4. Perform the experiment. When performing the experiment, turn on the spark timer just before the release button is pushed and turn it off as soon as the cylinder hits the clay disk.

Hints:

1. Be careful that the cylinder is not wobbling when it starts to fall.

2. High-voltage sparks can really give you a jolt. Be very careful not to touch any wires or the terminal on the timer when the spark timer is turned on.

3. After you obtain a set of dots on your paper, study them briefly to see if there are missing or extraneous dots. It is easier to repeat the experiment than to analyze poor data.

Fig. 1.(a) Diagrammatic illustration of the apparatus used to measure g.
(b) A typical set of data points
Group discussion and conceptual practice (These are not turned in; your practice these skills for the evaluation at the end of the lab).

G1. Discuss what the spacing of the dots means, and how it represents an object speeding up.
G2. From the dots, what was the total time it took the object to fall the length of the tape?
G3. Sketch what the spark dots would look like if an object speeds up and then slows down.
G4. Sketch what the spark dots would look like for a mass hanging from a spring, oscillating up and down in front of paper that is held still.

B. The Analysis
The next step is to interpret the information provided by the dots on the paper in terms of the mathematical description of the motion given by equations (1) and (2) for x(t) and v(t).

First, decide where to call x=0 and t=0. Most students would like to have x=0 and t=0 when v=0 (right when the cylinder starts to fall). The problem that arises from this is you don’t know anything about that instant because the cylinder is dropped independent of the time of any of the sparks; that is, the cylinder starts to fall sometime between two of the sparks. You could select the time of any spot as t=0, record x values for all t values after that and equation (1) and (2) would still hold. So choose a dot near the beginning, but not the big bunch of spots that represents where the cylinder was held still. To get the best results, you should take data over the largest span of the variables possible. Therefore, use dots all the way to the bottom.

1. Position vs time.
Measure the distance of the subsequent dots to this starting point. To give you a range of data points you might want to measure every second dot. Make a table of x vs t, remembering that every dot represents 1/60 of a second.

Produce a position versus time graph using Fit-Graph on the computer. Put the t list in the first (“x”) column on the computer, because you want that on the horizontal axis of the plot for x(t). Put the x data in the second (“y”) column. That way the fit will give x(t) instead of t(x). After you have a graph, perform a fit of a polynomial equation that best fits the data. Print this graph and attach it to your lab report. Record your equation and extract from it a value for g. Compare your measured values of g with the accepted value of 9.798 m/sec^2 in Provo. Calculate the percent error, which describes the fraction the error is of the expected value:

\[
\%\text{error} = \frac{\text{error}}{\text{expected value}} \times 100 = \frac{\text{experiment value} - \text{expected value}}{\text{expected value}} \times 100
\]

2. Velocity vs time
Suppose the points in Fig. 2 were spark spots. Since point b falls midway in time between points a and c, the average velocity of the cylinder between times a and c is the same as the instantaneous velocity right at the time spot b was made (this only works because v is increasing with a uniform acceleration, i.e. v(t) is a straight line). So by measuring the distance between the spots a and c, and dividing by the time

Fig. 2 Illustration of how instantaneous velocity can be determined.
interval between sparks a and c, you have the velocity when spark b was made. Obtain velocities for eight to ten specific times from the dots. Make a table and a plot of v(t) using **Fit-Graph** on the computer. After you have a graph, perform a fit of a line that best fits the data. Print this graph and attach it to your lab report. Record your equation and extract from it a value for g. Compare your measured values of g with the accepted value of 9.798 m/sec² in Provo. Calculate the percent error.

**Group discussion and conceptual practice:**

G8. Does the graph of v(t) you obtained look like what you expect? What is the value of v₀, and what is its meaning?

G9. Roughly what is the average velocity over this time, from the plot? Check it to see if it is the same as \( \frac{\Delta x}{\Delta t} \), where \( \Delta x \) is the total distance fallen in the total time you measured.

G10. Find the slope of v(t) line directly from the graph, and label on the graph \( \Delta v \) and \( \Delta t \). How does this relate to g and to the fit?

G11. From the total time, the distance covered and v₀ from your results, use the kinematic equations to find a value for the acceleration g. How does this compare with the values of g you get from a curve fit. Which method should be most reliable and why?

G12. For the figure at the right, which curve represents the motion with the largest average velocity? Which v(t) curve has an average that is the same as the v(t) found midway through the time interval, and why? Do they all have the same \( \Delta x \) over the time shown?

G13. Sketch x(t) for the middle v(t) curve, and then show how x(t) for the top curve might be different. Choose any \( x₀ \) you wish.

G14. Sketch a(t) for the three curves. What can you say about the average acceleration for the three curves?

G15. Sketch what v(t) and a(t) would look like if an object were speeding up and then slows down.

G16. Sketch what v(t) and a(t) would look like of a mass hanging from a spring, oscillating up and down.

G17. Sketch what v(t) would look like for a man jumping out of an airplane with a parachute. Sketch curves of x(t) and a(t) that might go with it. Choose whatever direction you wish as positive.
Group Report  
107 Lab 1 Motion in One Dimension

Names ______________________   ____________________________  
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Lab TA ______________________   Section______ Exp Score (0-10)____

B1. x(t) data

<table>
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<th>Dot number</th>
<th>t in seconds</th>
<th>x in meters from the x=0 dot</th>
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Equation of best fit:
x(t) =

Calculation of g from fit coefficients 
(show or explain your logic):

Percent error in g: 
(show the steps with units):

B2. v(t) data and analysis

Show sample calculation of v at some time t:

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<th>time t in seconds</th>
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Equation of best fit:
v(t) =

Calculation of g from fit coefficients:
(show or explain your logic):

Percent error in g:
(show the steps with units):

Attach your x(t) and v(t) graphs.

Discuss how well your results agreed with physical principles. If you did not finish the experiment or analysis, please comment on what went wrong. Do not write more than in the space below