Waves

Waves on a string
Will understanding how waves travel through strings attached to walls—a situation I imagine we will rarely encounter—help us understand how waves travel through other mediums, such as air? Also, will you briefly explain the homework late policy referenced in the syllabus?

- Yes! Just like understanding how objects move without friction or air resistance help you understand more realistic motion! (We have to start somewhere!)
- You can turn in four assignments late without penalty. After that, each late assignment is worth half credit.
- Do these concepts apply only to transverse waves?
  - Yes and No.
- PpP mentioned how one equation would be harder if certain assumed things weren't true. Will we deal with those situations in this class, later classes, or not at all?
  - We will discuss them briefly in this class. (One will be the subject of a lab.) You will probably see them more in later classes. Some of these things are areas of active research (in nonlinear acoustics for example).
What’s the difference between these:

- \( s = \cos(x - 5t) \)
- \( s = \cos(2(x - 5t)) \)

General form of cosine wave:

- \( s = A \cos(k(x - vt) + \phi) \)
- Sometimes written as:
  - \( s = A \cos(kx - \omega t) + \phi \)
  - \( \omega = kv \)

\( k = \text{“wave vector”}; \ \omega = \text{“angular frequency”} \)
What are the units of:
- $k$? $x$?
- $kx$?
- $w$? $t$?
- $wt$?

From Warmup: Can you explain a little bit more about what a phase is? Thanks!
The (Linear 1D) Wave Equation

\[ \frac{\partial^2 s}{\partial t^2} = v^2 \frac{\partial^2 s}{\partial x^2} \]

* Why is it called the wave equation?
* Because traveling waves are solutions of the equation!

\[ s = A \cos(x - vt) \]

\[ \frac{\partial s}{\partial x} = -A \sin(x - vt) \]

\[ \frac{\partial^2 s}{\partial x^2} = -A \cos(x - vt) \]

\[ \frac{\partial s}{\partial t} = -A \sin(x - vt)(-v) \]

\[ = +Av \sin(x - vt) \]

\[ \frac{\partial^2 s}{\partial t^2} = +Av \cos(x - vt)(-v) \]

\[ = -Av^2 \cos(x - vt) \]

\[ = v^2 \frac{\partial^2 s}{\partial x^2} \]

Any function that has “x – vt” will work! Try it!
Why is it called the **linear** wave equation?

* Because we don’t have nonlinear terms in the equation itself
* \( s^2, \frac{\partial s}{\partial x}, \frac{\partial s}{\partial t}, \ldots \)

Properties of linear differential equations

* If \( s_1 \) is a solution, then so is \( C \times s_1 \)
* If \( s_1 \) and \( s_2 \) are solutions, then so is \( (s_1 + s_2) \)
* Consider a medium with \( v = 3 \text{ m/s} \)
  
  \[
s = 2 \cos(2(x - 3t) + 5.5) - 4 \cos(9(x - 3t) + 0.3)
  \]

Is a perfectly acceptable wave!
Analysis: A section of rope

* (Chalkboard)
A wave pulse traveling on a string hits the end of the string, which is tied to a post. What happens?

a) The pulse reflects, flipped over
b) The pulse reflects, not flipped over
c) The post is violently ripped out of the ground and impales the onlookers.
What happens when two wave pulses on a linear medium run into each other head on?

a) They reflect off of each other and go back the way they came.

b) Part of each wave is reflected and part is transmitted

c) They pass right through each other
From warmup: String attached to a wall. Which of the following will decrease the time required for the pulse to reach the wall? Mark all that apply.

- Moving your hand up and down more quickly but by the same amount
- Moving your hand up and down more slowly but by the same amount
- Moving your hand the same speed but farther up and down
- Moving your hand the same speed but a shorter distance up and down
- Using a heavier string of the same length under the same tension
- Using a lighter string of the same length under the same tension
- Using the same string of the same length but under more tension
- Using the same string of the same length but under less tension

- Rubber tubing
- Web demo: http://www.colorado.edu/physics/phet/simulations/stringwave/stringWave.swf
A wave pulse traveling on a string meets an interface, where the medium abruptly switches to a thicker string. What happens?

a) The pulse continues on, but flipped over
b) The pulse continues on, not flipped over
c) The pulse reflects, flipped over
d) The pulse reflects, not flipped over
e) The pulse partially reflects and partially transmits
Power Energy Transfer

\[ P = \frac{1}{2} \mu \omega^2 A^2 v \]

* What does everything stand for?
* Proved in book; most important thing for us now is \( P \sim A^2 \)
* From warmup: Explain how each of the following (keeping everything else the same) would impact the energy transfer rate of a wave on a string:
  a) Reducing the mass density of the string
     - Reduces \( P \) linearly, assuming velocity is unchanged
  b) Doubling the wavelength
     - Irrelevant?
  c) Doubling the tension
     - Increases \( v \) and therefore \( P \) (by \( \sqrt{2} \))
  d) Doubling the amplitude
     - Increase by 4x