4. Consider a sphere of hot aluminum with a uniform initial temperature of 870 K and a radius of \( a = 0.05 \) m. The important thermal constants of aluminum are:

- **Specific heat:** \( c = 900 \text{ J/kg} \cdot \text{K} \)
- **Density:** \( \rho = 2.7 \times 10^3 \text{ kg/m}^3 \)
- **Thermal conductivity:** \( \kappa = 238 \text{ W/m} \cdot \text{K} \)
- **Emissivity:** \( e = 0.05 \)
- **Stefan-Boltzmann constant:** \( \sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \cdot \text{K}^4 \)

The thermal diffusion equation for aluminum in spherical geometry is:

\[
\rho c \frac{\partial T}{\partial t} = \frac{\kappa}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right)
\]

(a) First solve this diffusion problem with the fixed outer temperature boundary condition:

\[ T(a) = 293 \text{ K} \]

Find the time at which the central temperature \( T(0) \) reaches 325 K.

(b) Now solve this diffusion problem again, but with the aluminum suspended weightless in the vacuum of outer space so that it can only lose heat by radiation. The outer boundary condition in this case is:

\[-\kappa \frac{\partial T}{\partial r} = \sigma e T^4 \text{ at } r = a\]

Again, find the time at which the central temperature reaches 325 K, accurate to 3 significant figures. You will want to use the ability of Crank-Nicholson to take large time steps.