Physics 360 – Winter 2006
Exam #2

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CID: _____________

Score: ____________

- Calculators are not permitted
- Nothing written on the colored cover sheet from the testing center will be graded.
- Be sure your test has seven pages excluding the colored cover sheet.
- Each problem is worth 10 points

Section I. Review Question

1. In a PV diagram, the process A→B is constant volume, B→C is constant pressure, and A→C is isothermal. Find the change in enthalpy for each of the three processes. Assume that: 1)there are N molecules of ideal gas, 2) the gas has f degrees of freedom, and 3) you know P_A, V_A, T_A, and P_B. You may also use Boltzmann's constant, k, in your answers.

Constant V

$$dH = dU + VdP = Q + VdP$$
$$= C_V dT + VdP = \frac{f}{2} NkdT + VdP$$
$$VdP = NkdT$$
$$dH = \frac{f+2}{2} VdP$$
$$\Delta H = \frac{f+2}{2} V_A (P_B - P_A)$$

Constant P

$$dH = dU + PdV = Q - PdV + PdV$$
$$= C_P dT = \frac{f+2}{2} NkdT$$
$$PdV = NkdT$$
$$dH = \frac{f+2}{2} PdV$$
$$\Delta H = \frac{f+2}{2} P_B (V_C - V_B)$$
$$= \frac{f+2}{2} (P_A - P_B) V_A$$

Isothermal = sum of the other 2, or

$$dH = PdV + VdP = NkT\frac{dV}{V} + NkT\frac{dP}{P}$$
$$= NkT_A \left[\ln\frac{V_C}{V_A} + \ln\frac{P_C}{P_A} \right]$$
$$= NkT_A \ln\frac{V_C P_C}{V_A P_A} = NkT \ln 1 = 0$$

Note: $V_B = V_A$, $P_C = P_B$, $P_A V_A = P_C V_C$

Section II. Qualitative Questions.

2. Someone states that the Theory of Evolution must be discarded because it violates the Second Law of Thermodynamics. Explain why evolution either violates or does not violate the Second Law.

Entropy within a closed system cannot decrease; however, the closed system that includes living species also includes the earth and the sun. So as long as the entropy of larger system does not decrease, evolution does not violate the Second Law of Thermodynamics.

3. What is a reversible process? Give two examples of reversible processes and two examples of irreversible processes.

A reversible process can be defined in several ways. Two are: 1) The system can always be represented by a well defined point on a PV diagram. W and Q can be calculated. 2) The system can be played in reverse and make physical sense.

4. A box of volume V contains six helium atoms. Considering each atom as a point particle, how many dimensions are there in the phase space of the system (the six atoms)?

36 (half credit for 6).

Section III. Basic Problems

5. Assume that the volume of one state in phase space is h^3. Find the multiplicity for a single monatomic atom of mass m in a box of dimension a on a side and total kinetic energy U. Assume an uncertainty in energy of ΔU. (Your answer should include only the variables defined within the problem.)

$$\Omega = \frac{a^3 4\pi p^2 \Delta p}{h^3} = \frac{a^3 4\pi 2mUm\Delta U}{h^3\sqrt{2mU}} = \frac{4\pi a^3 m\sqrt{2mU}\Delta U}{h^3}$$

$$\text{Note:}\quad U = \frac{p^2}{2m} \Rightarrow \Delta U = \frac{p\Delta p}{m}$$

6. A box of volume V contains one mole of one ideal gas at STP (0° C and 1 atm). A second box of volume $2V$ contains one mole of a second ideal gas, also at STP. A small tube then joins the boxes so the gases can slowly pass between the two containers. Calculate the change in entropy after a long time has passed.

The process is not reversible, but the change in entropy is the same as for an isothermal process that takes each gas from its initial configuration to its final configuration.

$$\Delta S_1 = N_A k \ln\frac{3V}{V} = N_A k \ln 3$$

$$\Delta S_2 = N_A k \ln\frac{3V}{2V} = N_A k \ln\frac{3}{2}$$

$$\Delta S = N_A k \ln\frac{9}{2}$$

7. An Einstein solid with $N>>q$ and unit energy ε has entropy given by the expression:

$$S = k\frac{U}{\varepsilon}\left[1 + \ln\frac{N\varepsilon}{U}\right]$$

(A) Find an expression for T.
(B) Find an expression for C_V.
(C) Show that your expression satisfies the Third law of Thermodynamics.

$$\frac{1}{T} = \left(\frac{\partial S}{\partial U}\right)_{N,V} = \frac{k}{\varepsilon}\left[1 + \ln\frac{N\varepsilon}{U}\right] + \frac{kU}{\varepsilon}\left(-\frac{1}{U}\right) = \frac{k}{\varepsilon}\ln\frac{N\varepsilon}{U}$$

$$\ln\frac{N\varepsilon}{U} = \frac{\varepsilon}{kT},\quad U = N\varepsilon\, e^{-\varepsilon/kT}$$

$$C_V = \left(\frac{\partial U}{\partial T}\right)_V = N\varepsilon\, e^{-\varepsilon/kT}\left(\frac{\varepsilon}{kT^2}\right) = \frac{N\varepsilon^2}{kT^2}e^{-\varepsilon/kT}$$

As T goes to zero, the exponent goes to zero very rapidly, so C_v goes to zero.

8. For a typical system, entropy is a function of *U, V,* and *N*. From this information and the expressions you know for *T, P,* and μ, show that the Thermodynamic Identity follows. Use the Thermodynamic Identity to find a possibly useless relationship for P/μ. (Be sure to specify which variables are fixed in your expression.)

$$dS = \frac{\partial S}{\partial U}dU + \frac{\partial S}{\partial V}dV + \frac{\partial S}{\partial N}dN$$
$$= \frac{1}{T}dU + \frac{P}{T}dV - \frac{\mu}{T}dN$$
$$dU = TdS - PdV + \mu dN$$

$$\frac{P}{\mu} = \left(\frac{\partial N}{\partial V}\right)_{U,S}$$

Section IV. Synthesis Problems

9. A two-state paramagnet consisting of N_d dipoles interacts with an Einstein solid having N_o oscillators. The energy step size for the oscillators is ϵ. The magnetic moment of each dipole is μ (note that this is not the chemical potential). There is an external magnetic field, B.

The entropy of the paramagnet is: $S_d = N_d k \ln N_d - N_\uparrow k \ln N_\uparrow - (N_d - N_\uparrow) k \ln(N_d - N_\uparrow)$.

The energy of the paramagnet is: $U_d = \mu B(N_d - 2N_\uparrow)$

The entropy of Einstein solid is: $S_o = N_o k \ln q - N_o k \ln N_o + N_o k$

The energy of the Einstein solid is: $U_o = q\varepsilon$

Find an expression for U_d. Write your answer in terms of N_d, N_o, μ, B, and the total energy U. You do not need to solve for U_d. in that expression.

Hint: The systems are in thermal equilibrium.

$$\frac{\partial S_o}{\partial U_o} = \frac{\partial S_d}{\partial U_d}$$

$$\frac{\dfrac{\partial S_o}{\partial q}}{\dfrac{\partial U_o}{\partial q}} = \frac{\dfrac{\partial S_d}{\partial N_\uparrow}}{\dfrac{\partial U_o}{\partial N_\uparrow}}$$

$$\frac{\dfrac{N_o k}{q}}{\varepsilon} = \frac{-k \ln N_\uparrow - k + k \ln(N_d - N_\uparrow) + k}{-2\mu B}$$

$$\frac{-2\mu B N_o k}{q\varepsilon} = -k \ln\left[\frac{N_d}{2} - \frac{U_d}{2\mu B}\right] + k \ln\left[\frac{N_d}{2} + \frac{U_d}{2\mu B}\right]$$

$$\frac{2\mu B N_o}{U_o} = \ln \frac{N_d \mu B - U_d}{N_d \mu B + U_d}$$

$$\ln \frac{N_d \mu B - U_d}{N_d \mu B + U_d} = \frac{2\mu B N_o}{U - U_d}$$

10. If you toss three dice simultaneously, the total points range from 3 to 18. We can think of the total points as the "energy" of a system. (You may leave *k* and logarithms in your answers.)

(a) Finish filling out the table below:

(If I made any mistakes in filling out the table, you'll need to correct them.)

Total Points	Points on Individual Dice	Multiplicity	Entropy
3	111	1	*k ln1 or 0*
4	112	3	*k ln 3*
5	113 122	6	etc.
6	114 123 222	10	
7	115 124 133 223	15	
8	116 125 134 224 233	21	
9	126 135 144 225 234 333	25	
10	136 145 226 235 244 334	27	
11	146 155 236 245 335 344	27	
12	156 246 255 336 345 444	25	
13	166 256 346 355 445	21	
14	226 356 446 455	15	
15	366 456 555	10	
16	466 556	6	
17	566	3	
18	666	1	

(B) Two people play a game where one start with three ones on their dice and the other start with three threes, so the total score is 12. The first person rolls one die and then the second person adjusts their dice so the total score remains at 12. (If the first person's total is more than 9, then he must leave his score at 9, and the second person leaves his total as 3, so the sum remains at 12. Don't worry about how this changes the outcome, however.)

The following is a table of the joint probability for this system. Finish the table.

First Score	Second Score	Total multiplicity
3	9	25 (Write in this box the total number of ways the first score is three while the second score is nine.)
4	8	63
5	7	90
6	6	100
7	5	90
8	4	63
9	3	25

(C) What would correspond to temperature in the rolling of three dice? Estimate the "temperature" at which the two sets of dice would be in thermal equilibrium.

$$Temperature = \frac{1}{\left(\dfrac{Change\ in\ entropy}{Change\ in\ points\,(energy)}\right)}$$

(This applies to each system individually, not to the combined system.)

$$T \approx \frac{1}{k\ln\frac{10}{6}}\ or\ \frac{1}{k\ln\frac{15}{10}} = \frac{1}{k\ln\frac{5}{3}}\ or\ \frac{1}{k\ln\frac{3}{2}}\quad or\ some\ average\ of\ these\ numbers.$$