

Physics 360 Exam #1
Winter 2006

Instructor: Lawrence Rees

CID: _____

(Please do NOT put your name on this exam.)

Section I. Qualitative Questions

1. Describe the degrees of freedom of a diatomic gas. Which degrees of freedom are available at low temperature? at room temperature?

The answers I write are very minimal answers. Students should elaborate somewhat.

Translational (3), rotational (2) – the moment of inertia about the molecular axis is small enough that this doesn't contribute, vibrational (2) – kinetic and potential.

Only translational at low temperature, then rotational comes in at room temperature.

2. Which is larger C_v or C_p ? Use the First Law of Thermodynamics to explain why.

C_p is larger. If volume is constant, all the heat flowing in goes to internal energy and hence higher temperature. If pressure is constant, some heat goes into work.

3. Describe in words what enthalpy is. If you just state the equation for enthalpy, you will receive half credit.

The energy necessary to create something out of nothing plus the work necessary to move the atmosphere out of the way to make room for it. $H = U + PV$.

4. What is meant by the “fundamental assumption of statistical mechanics”?

All microstates have equal probability of being occupied.

Section II. Basic Problems.

5. N atoms of a monatomic ideal gas go from an initial pressure and volume P_A, V_A to a final volume V_B . Your answers may involve only these variables along with Boltzmann's constant, k .

(A) Assume the process is isothermal. Find an expression for the work done. (Use the book's sign convention for work.)

$$W = - \int p dV = - NkT_A \ln \frac{V_B}{V_A} = - P_A V_A \ln \frac{V_B}{V_A}$$

(B) Assume the process is at constant pressure. Find Q .

$$Q = C_p \Delta T = Nk \frac{f+2}{2} (T_B - T_A) = \frac{f+2}{2} (P_B V_B - P_A V_A) = \frac{5}{2} P_A (V_B - V_A)$$

6. A quantity of ideal gas has a number density (the number of molecules per unit volume) n . Consider each molecule to be sphere of radius a . Find an expression for the mean free path of one of these gas molecules.

$$n_{coll} = \frac{N}{V} \pi (2r)^2 \ell$$

$$1 = \frac{N}{V} \pi (2r)^2 \langle \ell \rangle$$

$$\langle \ell \rangle = \frac{V}{4 \pi r^2 N}$$

7. In a PV diagram, the process A→B is constant volume, B→C is constant pressure, and A→C is isothermal. Find the change in enthalpy for each of the three processes. Assume that: 1) there are N molecules of ideal gas, 2) the gas has f degrees of freedom, and 3) you know P_A, V_A, T_A , and P_B . You may also use Boltzmann's constant, k , in your answers.

<i>Constant V</i>	<i>Constant P</i>	<i>Isothermal = sum of the other 2, or</i>
$dH = dU + VdP = Q + VdP$ $= C_v dT + VdP = \frac{f}{2} Nk dT + VdP$ $VdP = Nk dT$ $dH = \frac{f+2}{2} VdP$ $\Delta H = \frac{f+2}{2} V_A (P_B - P_A)$	$dH = dU + PdV = Q - PdV + PdV$ $= C_p dT = \frac{f+2}{2} Nk dT$ $PdV = Nk dT$ $dH = \frac{f+2}{2} PdV$ $\Delta H = \frac{f+2}{2} P_B (V_C - V_B)$ $= \frac{f+2}{2} (P_A - P_B) V_A$	$dH = PdV + VdP = NkT \frac{dV}{V} + NkT \frac{dP}{P}$ $= NkT_A \left[\ln \frac{V_C}{V_A} + \ln \frac{P_C}{P_A} \right]$ $= NkT_A \ln \frac{V_C P_C}{V_A P_A} = NkT \ln 1 = 0$

Note: $V_B = V_A, P_C = P_B, P_A V_A = P_C V_C$

8. What is the fractional probability of getting 10 heads if you toss 20 fair coins? You may leave your answer in terms of factorials, but please reduce all combinations to factorials.

$$\frac{1}{2^{20}} C\binom{20}{10} = \frac{1}{2^{20}} \frac{20!}{10!10!}$$

9. An Einstein solid has 15 units of energy distributed among three oscillators. What is the fractional probability that at least one of the oscillators is in its lowest energy state? You may leave your answer in terms of factorials, but please reduce all combinations to factorials.

The total number of microstates is

$$C\binom{17}{15} = \frac{17!}{15!2!} = 17 \times 8 = 136$$

If one oscillator is in its lowest state (no units of energy), then:

*Number of units of energy
in each of the three oscillators
(sequence unimportant)*

Multiplicity

0,0,15	3
0,1,14	6
0,2,13	6
...	
0,7,8	6

$$\text{Total probability} = \frac{3 + 7 \times 6}{136} = \frac{45}{136}$$

III. Synthesis Problems

10. An icosahedral die has 20 identical sides, numbered 1 through 20. In three rolls of this die, what is the probability of the sum of the rolls equaling 6? Please reduce your answer to a simple fraction or decimal number.

Total number of microstates in three rolls is $20^3 = 8000$.

<i>Combinations leading to 6</i>	<i>Multiplicity</i>
1,1,4	3
1,2,3	6
2,2,2	1

$$\text{Total probability} = \frac{10}{8000} = \frac{1}{800}$$

11. Three weakly interacting Einstein solids each have six oscillators. There are 7 total units of energy distributed among these three solids. What is the probability that oscillator A has 3 units of energy, oscillator B has 2 units of energy, and oscillator C also has 2 units of energy? You may leave your answer in terms of factorials, but please reduce all combinations to factorials.

Note: I said among “three oscillators” when I meant “three solids.” I think they’ll mostly interpret it the way I intended, but in case they used 3 as the number of oscillators – grade it as fairly as you can, but I’m not sure the answer could make much sense.

$$\text{Total number of microstates} = C\left(\begin{matrix} 7 + 17 \\ 7 \end{matrix}\right) = \frac{24!}{7!17!}$$

Multiplicity of the macrostate specified:

$$C\left(\begin{matrix} 3 + 5 \\ 3 \end{matrix}\right) \times C\left(\begin{matrix} 2 + 5 \\ 2 \end{matrix}\right) \times C\left(\begin{matrix} 2 + 5 \\ 2 \end{matrix}\right) = \frac{8!7!7!}{3!5!2!5!2!5!}$$

The total probability is the ratio of the two.