Physics 321
Take-home Test 6

Problem 1.

A lamina is in the shape of a right triangle with the two sides (not the hypotenuse) with lengths a = 6.0 cm and b = 4.5 cm. The mass of the traingle is 250 g and the density is uniform. We choose the x-axis of a coordinate system to along the 6.0 cm side and the y-axis to be along the 4.5 cm side.

(A -20 points) Find the inertia tensor for the lamina.
Do not put numerical values in yet!
(B - 20 points) Find the principal axes and the moments of inertia about those axes.
(C - 10 points) Find the normalized eigenvecors, ev1, ev2, and ev3, that correspod to the directions of the principal axes. Take dot products (DotProduct) and cross products (CrossProduct) of pairs of these to see if the eigenvectors form a right-handed, orthogonal basis set, as expected. (Change the names of the eigenvectors so that they are right-handed, if necessary.)
Are these values what you expected? Explain:

Problem 2.

Early in the semester, we showed that a bicycle tire on an extended axle precesses when held by the end of the axle. In this problem we will model the bicylcle tire by a circular lamina of mass M = 3.55 kg and radius R = 40.0 cm. The density of the lamina is uniform. The lamina turns on an axle that extends to a distance of l = 35.0 cm on either side of the axle. We take the axle to be massless. We spin the tire until it attains an angular velocity of 50 rad/sec and then release it from rest (except for the rotation described) with the axle horizontal and fixed at one end.

(A - 10 points) Find I33 and I11.
You may find the parallel axis theorem and the structure of the inertia matrix useful to save computation.
(B- 20 points) Write down the Lagrangian and find the equations of motion for the system. Solve the system for the boundary conditions indicated.
(C -15 points) By graphical means, find the period for the precession of the wheel (the time it takes to go through an angle of phi = 360 degrees).
Also find the period of the nutation (the time it takes to go from an angle of theta_max back to an angle of theta_max).
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Finally, find the period of rotation (the time it takes to go through an angle of psi; = 360 degrees).
Hint: Be sure to include numpoints=large number (a few thousand is even appropriate!) if you need to.
(D - 5 points) Describe the motion of the wheel.