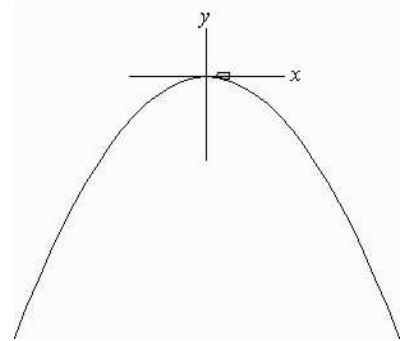


Physics 321
Take-home Test 4

Problem 1.

A small block is allowed to slide frictionlessly from rest down a parabolic dome. Call the height of the dome y and assume the motion of block is in the x - y plane. The shape of the dome is given by the equation $y = -\alpha x^2$.



You are to find the force on the block from the dome by using Lagrange multipliers.

Let

$$m = 100 \text{ g}$$

$$\alpha = 3.00$$

$$x_0 = 1.00 \text{ mm (the } x \text{ position of the block when it is released)}$$

(A - 10 points) Find the Lagrangian (using the variables x and y).

(B - 10 points) Using a Lagrange multiplier, find the equation of constraint and the equations of motion.

(C - 10 points) Use these three equations to find the equation of motion for $x(t)$.

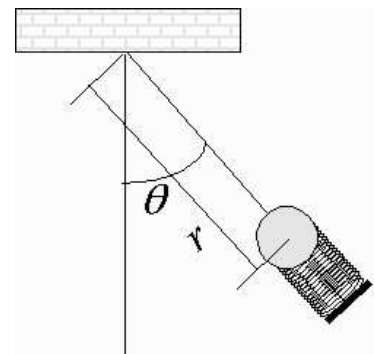
(D - 5 points) Put in values for the constants, and solve for $x(t)$ numerically. Plot the solution.

(E - 10 points) Solve for F_x and F_y , the forces of constraint in the x and y directions.

(F - 5 points) Plot the kinetic energy, the potential energy, and the total energy as a function of time on one plot. (You'll need to rewrite T and U in terms of the solution to the differential equation to be able to plot these quantities.)

Problem 2.

A mass m can slide freely on the rod a pendulum. A spring is placed between it and a small plate attached to the end of the rod, as shown in the diagram. Consider the rod, spring, and plate to be massless. The distance from the pivot point to the center of the mass is r . r_0 is the distance of the mass from the pivot when the pendulum is horizontal ($\theta = \pi/2$). The rod makes an angle θ with respect to the vertical, with $\theta = 0$ taken to be downward. The spring has a spring constant, k .



Let:

$$m = 500 \text{ g}$$

$$k = 20.0 \text{ N/m}$$

$$r_0 = 1.00 \text{ m}$$

(A - 15 points) First we want to solve the problem using the Lagrangian approach. Find the Lagrangian, the equations of motion, and plot $x(t)$ and $\theta(t)$.

(B - 35 points) Now solve the same problem using a Hamiltonian. Use the names r , θ , p_r , and p_θ for the variables.