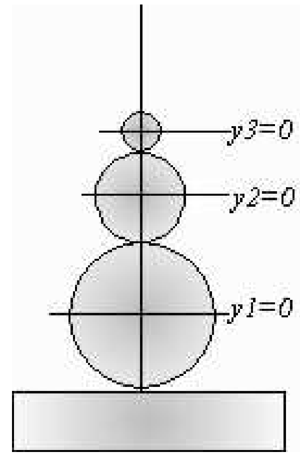
Physics 321
Weekly Test 2

Problem 1



Three balls each have a thin hole drilled through their diameter. The balls are then placed, largest first on a vertical rod as shown in the figure to the right.

The masses of the balls are 10 kg, 5 kg, and 1 kg. Find the height to which each of the three balls bounce after colliding with the ground. The height of each ball is measured with respect to its height when it is at rest. Thus $y1=y2=y3=0$ when all the balls are at rest. (Or equivalently, think of each ball as a point particle.) The balls are raised to a height of 1 meter (the bottom of the bottom ball is 1 m above the ground) and released. Assume that all collisions are elastic.

Hint: Think of the process as a series of collisions. The lower ball first collides with the ground and bounces upward into the second ball that is still going downward. The second ball then bounces off the first ball and collides, going upward, with the third ball that is going downward. Finally the first ball, which is going downward after the collision with second ball, collides with the ground a second time.

You may consider the mass of the earth to be infinite.

(A - 10 points) Find the momentum and kinetic energy of each ball just before any collisions have occurred. Take the positive direction to be upward.
Calculate the total kinetic energy and momentum before the collision. We will use this for comparison later.

(B - 5 points) Find the momentum and energy of ball 1 after it collides with the ground. (Call these p1a and k1a)

(C - 7 points) Find the momenta and energies of ball 1 and ball 2 after they collide. (Call these p1b, k2b, etc.) Then find the total kinetic energy and momentum of all three balls after this collision.

(D - 8 points) Find the momenta and energies of ball 2 and ball 3 after they collide. (Call these p2c, k3c, etc.) Then find the total kinetic energy and momentum of all three balls after this collision.

(E - 5 points) Finally, note that mass 1 was headed downward after its collision with mass 2. It then collides once more with the ground. Find its final energy and momentum. Call this k1d and p1d.

(F -10 points) Find the velocities of each ball and the heights to which each ball rises.

(G - 5 points) Find the total kinetic energies and momenta just after the last collision. Is total kinetic energy conserved? If not, why not? Is total momentum conserved? If not why not?

Problem 2

In Example 4.7 in the book, the potential energy of a rectangular prism of thickness *2b* placed on a cylinder of radius *r* is given by the expression:
$U(\theta) = mg[(b+r)\cos(\theta)+r\theta\sin(\theta)]$.

Find the period of oscillation of a block of mass *m*, length *L*, width *w*, and height *2b* rocking on the cylinder. Assume that the oscillations are small.

(A - 15 points) Find the moment of inertia of the block about an axis that is located at the bottom center of the block. (Note that the actual axis of rotation shifts a little, but for small oscillations, we can ignore this.)

(B - 10 points) Write the kinetic and potential energies of the block in terms of theta; and its time derivative. Remember to write $\theta(t)$ so you can take time derivatives.

(C - 15 points) Using $dT/dt + dU/dt=0$, find $\theta(t)$. [Note that dT/dt is diff(T(theta),t), not diff(T(theta(t)),t)] .
Assume the book is rotated to an angle $\theta_0 = 10$ degrees and then released from rest.

(D - 10 points) You could just estimate the period from the graph; however, I want you to do something different. Copy eqB2 in the Maple worksheet and edit it in two ways:
1) There is an overall factor of $d\theta / dt$ in the differential equation that can be eliminated. (If left in, Maple just solves $d\theta / dt = 0$.)
2) Make a small angle approximation. That is, substitute theta; for sin(θ) and 1 for cos (θ) in the differential equation.
Then solve the equation *non-numerically* and deduce the period from the solution.