Review for Test #1

1) Philosophical foundations
   a. Know Newton’s laws of motion, their meaning, and their validity

2) Single point particles in force fields
   a. Know $\vec{F} = m\vec{g}$, $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$, drag forces.
   b. Write equations of motion, specify boundary conditions
   c. Write a second-order DE as two first order equations
   d. Reproduce HW 2_1 for a few steps by hand

3) Interaction between point particles
   a. Find the center of mass – internal forces do not affect the center of mass motion
   b. Write collision equations – know the four types of collisions and what laws apply to each

4) Rotational motion
   a. Know how rotational variables and equations relate to linear variables and equations
   b. Know how to find angular momenta and torques
   c. Know how to apply conservation of angular momentum
   d. Know how to qualitatively analyze the motion of systems such as a bicycle wheel

5) Extended objects
   a. Write the equation for the center of mass
   b. Write the equation for the moment of inertia
   c. Write equations of motion for inclined planes, rolling objects, pulleys, strings, and pendulums

6) Energy
   a. Know the definitions of work and potential energy in terms of a line integral
   b. Know that for potential energy to be defined, a force can have no time nor velocity dependence and must have zero curl
   c. Know how to evaluate the curl of simple forces in Cartesian coordinates
   d. Be able to find $T$ and $U$ for the systems of 5(c) above
   e. Analyze the motion of a system if the potential energy function is given
   f. Write equations of motion using either $T + U = E$, $\dot{T} + \dot{U} = 0$.

7) Oscillations
   a. Know the general oscillator equation: $\ddot{x} + 2\beta \dot{x} + \omega_0^2 x = f(t)$
   b. Describe underdamped, overdamped, and critically damped oscillators. Know how each relates to the parameters of the general equation.
   c. Know that the solution for weakly damped oscillators is essentially the undamped oscillation multiplied by $e^{-\beta t}$.
   d. Describe the transient and the steady-state behavior of driven oscillators.
   e. Know the FWHM of the resonance curve is approximately $2\beta$.
   f. Q value:
      i. $Q = \pi \times \text{number of periods in one time constant (1/\beta)}$
      ii. $Q = 2\pi \times \text{energy/energy lost per cycle}$
      iii. $Q = \omega_0/(2\beta)$, “sharpness” of the resonance peak
Sample Questions

1. Write the second order differential equation \( \frac{d^2 y}{dx^2} = A \sin(x) \frac{dy}{dx} + B \cos(x) \) as two first order differential equations. Write a set of boundary equations that could be used to solve the equations numerically. (There is no unique set of boundary conditions that must be chosen, of course.)

2. We know that \( \frac{dy}{dx} = 3x^2 \). Using a step size \( \Delta x = 0.10 \), fill out the following table.

<table>
<thead>
<tr>
<th>x</th>
<th>y</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>0.1</td>
<td></td>
</tr>
</tbody>
</table>

3. State Newton’s three laws of motion. Briefly explain the meaning of each law. Describe one case where Newton’s third law does not hold.

4. You are given the following problem:
   A ladder leaning against a wall starts to slide. Find the motion of the ladder using forces and Newton’s second law. Explain how you would solve this problem. Be specific enough that I could follow your steps to complete the problem. The problem is rather complicated, so you not try to solve it – although it is good practice.

5. Write down the equations for the motion of a baseball in two dimensions. Include linear and quadratic drag terms.

6. What is the Lorentz force? Write down equations of motion for a proton moving in the magnetic field \( \vec{B} = B_0 \hat{z} \). Write down a reasonable set of boundary conditions that you could use to solve a specific problem in Mathematica. (You don’t need to use Mathematica notation.)

7. A proton and an antiproton collide and annihilate each other producing new particles. (An antiproton has the same mass as a proton, but a negative charge.) The kinetic energy of each particle is \( T_0 \) when the particles are far from each other. If two particles of equal mass are created in the process, what is the maximum mass they could each have? (Call the proton mass \( m \) and ignore any relativistic effects.) How does this answer differ if an antiproton of kinetic energy \( 2T_0 \) strikes a proton at rest?

8. After starting a bicycle tire spinning by hand, some mud flies off the outside edge of the tire. What happens to the angular speed of the tire after the mud leaves? Why?

9. A suitcase contains a spinning flywheel with its axis horizontal and pointing to your right as you walk straight forward. You make a sharp right turn. What happens to the suitcase? be specific. Explain why.
10. A lamina is a thin, flat sheet of material. An example would be a circle cut out of a sheet of cardboard. A square lamina is rotated about an axis passing through two corners as illustrated. Argue that if the thickness of the lamina, \( t \), is small, we can evaluate the moment of inertia by using the formula:

\[
\int d^2\rho dV = \int d^2\sigma dA
\]

where \( \sigma = \rho/t \) is called the areal density of the material. What does \( d^2 \) in the integral mean? If the square has sides of length \( a \), what is the moment of inertia about the axis shown.

11. Describe two methods we use to obtain differential equations by using conservation of energy. Apply each method to a simple pendulum in the small angle (\( \sin \theta = \theta \)) approximation.

12. A mass attached to a vertical spring is at equilibrium when \( y = 0 \). The spring constant is \( k \) and the mass is \( m \). The spring is massless.
   (a) Write the potential energy of the system in terms of \( y \) and \( y_0 \) where \( y_0 \) is the \( y \) position of the bottom of the spring when the mass is removed.
   (b) Find a value for \( y_0 \). Hint: This is the amount the spring stretches.
   (c) Take \( k = 1 \) and \( mg = 1 \) in SI units. (Yes, this is a very long but massless spring with a very small mass on it!) Sketch three curves on the graph below: 1) the potential energy of the spring, 2) the gravitational potential energy, and 3) the total potential energy.

\[
U \text{(J)}
\]

\[
\begin{array}{cccccc}
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 & & & & & \\
-2 & -1 & 0 & 1 & 2 & \\
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 & & & & & \\
 & & & & & \\
\end{array}
\]

(d) When the maximum kinetic energy is 0.5 J, the mass will oscillate between what values of \( y \)?

13. Describe the three types of harmonic oscillator damping. Write the differential equation for a damped harmonic oscillator. How is \( \beta \) related to \( \omega_0 \) for each case?
14. An underdamped harmonic oscillator is characterized by a certain $\beta$ and $\omega_0$. What is the time constant for the damping (the time it takes the amplitude to decrease by $1/e$)? At what frequency does the system oscillate? Hint: $e^t$.

15. Sketch the response $A^2$ of a driven harmonic oscillator as a function of the driving frequency $\omega$. At what frequency is the response a maximum? What does FWHM mean? What is the expression we use for the FWHM for $A^2$?

16. Define the $Q$ value of a damped harmonic oscillator (not driven), and of a driven harmonic oscillator. How is $Q$ related to $\beta$?