1. A sphere hangs by a massless spring from the ceiling. The linear drag coefficients for the sphere are \( c_1 \) and \( c_2 \) (these are not functions of the ball diameter). Let \( y = 0 \) correspond to the bottom of the spring when the mass is not attached.

(a – 5 points) Write the equation of motion including linear and quadratic drag terms.

\[
m\ddot{y} = -mg - c_1 \dot{y} - c_2 |\dot{y}| \dot{y} - ky
\]

(b – 5 points) Let \( z = y + a \) so that you can eliminate the gravitational term from the equation. What value does \( a \) have?

\[
y = e^{-a} \quad \text{so} \quad a = \frac{mg}{k}
\]

(c – 5 points) Find the corresponding characteristic (algebraic) equation.

\[
mr^2 = -c_1 r - c_2 |r| r - k
\]

(d – 5 points) Find the frequency of oscillation of the sphere when you ignore quadratic drag.

\[
\omega = \sqrt{\omega_0^2 - \beta^2} = \sqrt{\frac{k}{m} - \frac{c_1^2}{4m^2}}
\]

\[
r^2 + \left(\frac{c_1}{m}\right)r + \left(\frac{k}{m}\right) = 0 \quad \Rightarrow r = \frac{-2\beta \pm \sqrt{4\beta^2 - 4k}}{2}
\]

\[
r = -\beta \pm i\sqrt{\omega_0^2 - \beta^2}
\]

\[
z = e^{-\beta t} \times [A \cos \omega t + B \sin \omega t]
\]

\[
\omega = \sqrt{\omega_0^2 - \beta^2}
\]