Physics 321

Hour 6
Conservation of Angular Momentum

Linear and Angular Relationships

\[
\begin{align*}
\mathbf{p} &= m \mathbf{v} \\
F &= m \mathbf{a} \\
\mathbf{F} &= \mathbf{p} \\
v &= \dot{x} \\
a &= \ddot{x} \\
\ell &= \mathbf{I} \omega \\
\Gamma &= I \alpha \\
\mathbf{r} &= \mathbf{r} \times \mathbf{p} \\
\mathbf{F} &= \mathbf{r} \times \mathbf{F} \\
\ell &= l \omega \\
\Gamma &= \Gamma \alpha \\
\mathbf{r} &= \mathbf{r} \\
\omega &= \dot{\theta} \\
\alpha &= \ddot{\theta}
\end{align*}
\]

Angular Momentum in Linear Motion

We must calculate angular momentum about a point \( P \).

\[
\ell = \mathbf{r} \times \mathbf{p} \\
\ell = b \mathbf{p}
\]

Central Force

The torque is zero, so angular momentum is constant.

Kepler’s 2nd Law

The area of a trapezoid with sides \( \mathbf{A} \) and \( \mathbf{B} \) is

\[
\text{Area} = \left| \mathbf{A} \times \mathbf{B} \right|
\]

\[
\frac{dA}{dt} = \frac{1}{2} \frac{\left| \mathbf{r} \times \mathbf{v} \right| dt}{dt} = \frac{\ell}{2m}
\]

“Bomb” Problem

An artillery shell explodes just as the shell reaches its maximum height. It breaks into three pieces. Ignore drag forces and the mass of the explosive itself. Describe what happens.
Center of Mass
\[ \sum \vec{F}_{\text{ext}} = \sum m_i \vec{r}_i \equiv M\ddot{\vec{R}} \]

Moment of Inertia
\[ l \equiv \sum m_i d_i^2 \]

The Rocket Problem
Newton’s 2nd Law:
\[ \vec{F} = \vec{p} = m \vec{v} + m \vec{a} \]
This is true – but momentum conservation is more transparent:
Start with mass \( m \) and \( \vec{v} = 0 \). Let \( v_{\text{ex}} \) be the exhaust velocity.
\[
(m - |\Delta m|) \Delta \vec{v} = |\Delta m| (v_{\text{ex}} - \Delta \vec{v}) \\
m \Delta \vec{v} = v_{\text{ex}} \Delta m \\
m \vec{a} = v_{\text{ex}} |\vec{v}| \\
\]
Adding any external forces: \( \vec{F} = m \vec{a} = F_{\text{ext}} + v_{\text{ex}} |\vec{v}| \)
Thrust = \( v_{\text{ex}} |\vec{v}| \)