Physics 321

Hour 37
Collisions in Three Dimensions

Collision in the CM Frame
• Assume we know the masses, the initial momentum of each body (they’re the same), and the scattering angle.

Collision in the CM Frame
• Assume we know any energy loss as well.

Collision in the CM Frame
• First, we find $P$.

Collision in the CM Frame
• Then we find $T_1$ and $T_2$.
• Now we know everything!

Elastic Scattering
• If $E_{ex} = 0$, $P = \sqrt{2\mu(T_{01} + T_{02})} = P_0$

Elastic Scattering
• All the scattering can do is change the directions of the particles.

The CM Frame
• The cm frame is really easy...

The CM Frame
• Theorists (almost) always work in the cm frame.
• Experimentalists usually convert cross sections to the cm as well.
• But, experiments are not usually conducted in the cm frame!
Now let’s assume we have the CM solution and we want to get back to the lab frame.

- We know $P_0 = m_2 V_{20}$

$$m_1 \begin{bmatrix} 0 & 0 \\ V_{10} + V_{20} & 0 \end{bmatrix} + m_2 \begin{bmatrix} 0 & 0 \\ V_1 \sin \theta & 0 \end{bmatrix} = m_1 \begin{bmatrix} 0 & -V_2 \sin \theta \\ V_1 \cos \theta + V_{20} & 0 \end{bmatrix} + m_2 \begin{bmatrix} 0 & -V_2 \sin \theta \\ -V_2 \cos \theta + V_{20} & 0 \end{bmatrix}$$

So we can find the lab quantities $m_1 \begin{bmatrix} 0 \\ V_{10} + V_{20} \end{bmatrix} + m_2 \begin{bmatrix} 0 \\ V_1 \sin \theta \end{bmatrix} = m_1 \begin{bmatrix} 0 \\ V_1 \cos \theta + V_{20} \end{bmatrix} + m_2 \begin{bmatrix} 0 \\ -V_2 \cos \theta + V_{20} \end{bmatrix}$

• But usually we don’t know $\theta$ a priori.

Then we can find $P$ and everything else.

$$P_0 = \frac{\mu}{m_1} p_0 \quad P \sin \theta = p_1 \sin \theta_1$$

$$P \cos \theta = p_1 \cos \theta_1 - \frac{m_1}{m_2} P_0$$

$$P = \sqrt{2 \mu (T_{01} + T_{02} - E_{\text{ex}})} \quad \frac{P \sin \theta}{P \cos \theta + \frac{m_1}{m_2} P_0}$$

$$P_1^2 = (P \sin \theta)^2 + \left( P \cos \theta + \frac{m_1}{m_2} P_0 \right)^2$$

This is a quadratic equation we can solve for $P_1$.

Then we can find $\theta$ and everything else.