What Is Chaos?
If we write down a Lagrangian with fixed initial conditions, Mathematica will solve the problem exactly the same way every time. So is any motion really chaotic?

Chaos is when...
• Motion is non-repetitive
• Small changes in initial conditions lead to large changes in subsequent behavior

What Equations Are Chaotic?
• Equations must be nonlinear
  In linear equations each term includes $f$ or derivatives of $f$ linearly
• The nonlinear equations or initial conditions must be “complicated”

Successive Approximations
• One method of solving nonlinear equations is the method of successive approximations
• This isn’t the best way to solve chaotic systems...

Example
chaos_succ_app.nb

Analysis Techniques
• State space (or phase space) plots
• (Many others – bifurcation plots, Poincaré sections, ...)

Can plot $q$ vs $\dot{q}$.
Can plot $\sqrt{T} = \frac{p}{\sqrt{2m}}$ vs $\sqrt{U} = \sqrt{\frac{k}{2}} q$
then the plot is circular since $T+U=E$. 
Example
chaos_statespace.nb