Understanding the “Fictitious” Force

Think of \( r \) like \( x \):

The force acts in the \( +r \) direction.

Before we even consider real forces, we have to “add” a repulsive centrifugal force or centrifugal potential to the problem:

\[
F = \frac{l^2}{mr^3}
\]

\[
U = +\frac{l^2}{2mr^2}
\]
Centrifugal Potential

\[ u_{\text{cent}} = \frac{\hat{r}^2}{2m^2} \]

\[ u_g = -G \frac{Mm}{r} \]

Central Force

- Force is in the radial direction in spherical coordinates
- The curl is always zero – even if the force isn’t inverse square
- Source at the origin
  \[ \vec{F} = \frac{\alpha}{r^n} \hat{r} = \frac{\alpha \hat{r}}{r^{n+1}} \]
- Source at \( \vec{r}_1 \)
  \[ \vec{F}_{21} = \frac{\alpha (\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^{n+1}} \]
- Potential energy
  \[ U(r) = \frac{\alpha (n - 1)}{|\vec{r}_2 - \vec{r}_1|^{n-1}}, \quad n \neq 1 \]

Inverse-square Force

- Source at \( \vec{r}_1 \)
  \[ \vec{F}_{21} = \frac{\alpha (\vec{r}_2 - \vec{r}_1)}{|\vec{r}_2 - \vec{r}_1|^3} \]
- Potential energy
  \[ U(r) = \frac{\alpha}{|\vec{r}_2 - \vec{r}_1|} \]

Example

moon_orbit.nb