Physics 321

Hour 11
Simple and Damped Harmonic Oscillators

Bottom Line
- Equation for a damped oscillator:
  \[ m \ddot{x} = -kx - b \dot{x} \]
  \[ \ddot{x} = -\omega_0^2 x - 2\beta \dot{x} \]
- Damping regimes:
  - Underdamped: \( \beta < \omega_0 \)
  - Critically damped: \( \beta = \omega_0 \)
  - Overdamped: \( \beta > \omega_0 \)

Simple Harmonic Oscillator
1) What is the equation of motion?
2) Where is your origin?
3) What is the frequency of oscillation?
4) What are the three ways you can write the general solution?
5) How many constants do you have to fix to get a specific solution?
6) How do you find those constants?

Mass on a Massless Spring
1) Find \( U \) and \( T \)
2) \( \dot{T} + \dot{U} = 0 \)
3) Equilibrium
4) Equation of motion
5) \( \omega, T \)
6) Energy plot

Exam #1 Reminders (Spring 2015)
See Review #1, Sample Test #1, Example Problem
Review is Wednesday 5/13
Test is available Thurs. 5/14 – Monday 5/18
Average time will be about 1 hr 15 min

Example
VertSpringUWell.nb
A Shallow Frictionless Bowl

1) Assume $\dot{z} \approx 0, \ z = \alpha r^2$
2) Find $U$ and $T$
3) $\dot{T} + U = 0$
4) $\vec{F} = -\nabla U = m\ddot{z}$
5) $\omega, T$

Another Bowl

1) Assume $\dot{z} \approx 0, z = ax^2 + \beta y^2$
2) Find $U$
3) $\vec{F} = -\nabla U = m\ddot{z}$
4) Two frequencies

Example

Lissajous.nb

Damped Oscillator

The equation: $m\ddot{x} = -kx - b\dot{x}$

A little rearranging: $\ddot{x} = -\frac{k}{m}x - \frac{b}{m}\dot{x}$

$\ddot{x} = -\omega_0^2 x - 2\beta \dot{x}$

A trial solution: $x(t) = e^{rt}$

$r^2 x(t) = -\omega_0^2 x(t) - 2\beta r x(t)$

$r^2 + \omega_0^2 + 2\beta r = 0$

$r = -\beta \pm \sqrt{\beta^2 - \omega_0^2}$

Three Regimes

- Underdamped: $\beta < \omega_0$
- Critically damped: $\beta = \omega_0$
- Overdamped: $\beta > \omega_0$

Underdamped

$\beta < \omega_0, \ r = -\beta \pm \sqrt{\beta^2 - \omega_0^2}$

$r = -\beta \pm i\sqrt{\omega_0^2 - \beta^2}$

Solution:

$x(t) = Ae^{-\beta t} e^{i\sqrt{\omega_0^2 - \beta^2} t} + Be^{-\beta t} e^{-i\sqrt{\omega_0^2 - \beta^2} t}$

or

$x(t)$

$= Ae^{-\beta t} \sin(\sqrt{\omega_0^2 - \beta^2} t)$

$+ Be^{-\beta t} \cos(\sqrt{\omega_0^2 - \beta^2} t)$

Note: $\omega = \sqrt{\omega_0^2 - \beta^2}$
Overdamped

\[ \beta > \omega_0, \quad r = -\beta \pm \sqrt{\beta^2 - \omega_0^2} \]

Solution:

\[ x(t) = Ae^{-\beta t} e^{\sqrt{\beta^2 - \omega_0^2} t} + Be^{-\beta t} e^{-\sqrt{\beta^2 - \omega_0^2} t} \]

Critically Damped

\[ \beta = \omega_0, \quad r = -\beta \pm \sqrt{\beta^2 - \omega_0^2} \]

Solution:

\[ x(t) = Ae^{-\beta t} + Bte^{-\beta t} \]

Example

DampOsc5_4.nb