8.1 \[ T = \frac{1}{2} \text{me}^2 \dot{\theta}^2 \]
\[ U = -mgl \cos \theta \]
\[ \cos \theta \approx 1 - \frac{1}{2} \theta^2 \]
\[ U \approx \frac{1}{2} mgl \theta^2 \]
\[ T + U = E_0 \Rightarrow \frac{1}{2} \text{me}^2 \dot{\theta}^2 + \frac{1}{2} mgl \theta^2 = E_0 \]
\[ \Rightarrow \dot{\theta}^2 + mgl \theta = 0 \]
\[ \text{These can be simplified.} \]
3. (20 points) A mass attached to a vertical spring is at equilibrium when $y = 0$. The spring constant is $k$ and the mass is $m$. The spring is massless.

(a) Write the potential energy of the system in terms of $y$.

$$ U = \frac{1}{2} k(y - y_0)^2 + mg y $$

(b) Find a value for $y_0$, the $y$ position of the bottom of the spring when the mass is removed. Hint: This is the amount the spring stretches.

$$ -mg = -ky_0 $$

$$ U'(0) = 0 \Rightarrow k(-y_0) + mg = 0 $$

$$ y_0 = \frac{mg}{k} $$

(c) Take $k = 1$ and $mg = 1$ in SI units. (Yes, this is a very long but massless spring with a very small mass on it!) Sketch three curves on the graph below: 1) the potential energy of the spring, 2) the gravitational potential energy, and 3) the total potential energy.

(d) When the maximum kinetic energy is 0.5 J, the mass will oscillate between what values of $y$?

-1 to 1