

Physics 321
Homework 9

Due at midnight on the day of Hour 10.

- Conservation of energy can be used in two ways to derive equations of motion for a system:

A. $T + U = E$. For free fall, for example, this leads to:

$$\frac{1}{2} m \dot{x}^2 + mgx = E$$
$$\dot{x}^2 = \frac{2(E - mgx)}{m}$$

B. $\dot{T} + \dot{U} = 0$. Be careful that you correctly identify the total kinetic energy and potential energy of the system.

Problems

1. A small bar of soap with mass $m = 125$ g is placed in a bowl of hemispherical shape. The soap can slide frictionlessly on the surface of the bowl. The radius of curvature of the bowl is $R = 50.0$ cm. Use a coordinate system where the z axis points upward. (Thus, when the soap is in the bottom of the bowl $\theta = \pi$.) The soap has no component of velocity in the ϕ direction; that is, it slides only “up” and “down” the bowl. Let the bar of soap be initially at an angle of $\theta = 5\pi/6$ (30° up from the bottom of the bowl), and be moving up the bowl with a velocity of 2.00 m/s.

- Write down $T(v_\theta)$ and $U(\theta)$.
- Algebraically (not graphically), find the classical turning points.
- Make a plot showing $U(\theta)$ and E .

2. Write down equations for T and U for the same situation as in Problem 9.1. This time, however, make the following substitutions so that we can differentiate these functions with respect to time:

- in T change v_θ to $\text{diff}(\theta(t), t)$
- in U change θ to $\theta(t)$

Use the equation $\dot{T} + \dot{U} = 0$ to solve for $\theta(t)$.

Problems 3 & 4 worked in Maple 7, but no longer do in Maple 9/10... However, it is instructive to look at the solutions. – You need not turn them in.

3. Now repeat Problem 2, but with the initial velocity $\vec{v} = 1.414 \text{ m/s } \hat{\theta} + 1.414 \text{ m/s } \hat{\phi}$. Note that the magnitude of the velocity is the same as in Problem 2, but the direction is different.

4. Redo Problem 2, but use the equation in A above.