

Physics 321
Homework 8

Due at midnight on the day of Hour 9.

The important concepts from Hours 7 and 8 are:

1. We can define a potential energy only under some quite restrictive conditions.

A. The force must be a function of position only. It cannot have velocity or time dependence. (It can be a constant.)

B. The curl of the force must be zero. If this is true, the line integral around any closed path is zero.

2. The gradient

A. Operates on a scalar field and gives us a vector field.

B. Tells us how much and in what direction the field increases most rapidly.

C. In Cartesian coordinates, we may write

$$\nabla \psi = \frac{\partial \psi}{\partial x} \hat{x} + \frac{\partial \psi}{\partial y} \hat{y} + \frac{\partial \psi}{\partial z} \hat{z}.$$

3. $\vec{F} = -\nabla U$ if a potential energy exists.

4. The curl:

$$\nabla \times \vec{A} = \begin{vmatrix} \hat{x} & \hat{y} & \hat{z} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix} \quad (\nabla \times \vec{A})_x = \lim_{\Delta A \rightarrow 0} \frac{\oint \vec{A} \cdot d\vec{\ell}}{\Delta A} \Big|_{\text{path in } y-z \text{ plane}}$$

1. Use the form $F = \frac{\alpha}{r^2}$ for the magnitude of the force.

2. In cylindrical coordinates, the magnitude of a force is given by the expression $F = \frac{\alpha}{r^2}$. Is this a conservative force if the direction of the force is radial? tangential?

3. Find the force for each of the following potentials. Greek letters, other than the angles in cylindrical or spherical coordinates, are constants.

(A) $U = -\frac{\alpha}{r^3}$ in spherical coordinates.

(B) $U = \alpha \sin(\theta)$ in cylindrical coordinates.