Physics 321
Homework 6

Due at midnight on the day of Hour 7.

We will just do a few exercises in finding centers of mass, total masses, moments of inertia, etc.

The center of mass is generally given bu the expression:

$$m = \int \rho dV, \quad \vec{r}_{cm} = \frac{1}{m}\int \vec{r}\rho dV = \frac{1}{m}\int (x\hat{x} + y\hat{y} + z\hat{z})\rho dx dy dz$$

The important thing you need to remember in finding the moment of inertia of an object is that in the formula

$$I = \int r^2 \rho dV$$

the *r* may not be the *r* that is found in a given problem. – Slice the object into small volumes of mass $dm=\rho dV$. Then *r* is the distance from the rotation axis to this small volume.

1. For the following objects, find the total mass and the center of mass. α and β are constants in all that follow.

(A) A rod of linear mass density λ = α*x*^2; that lies on the *x* axis from -*L/2* to +*L/2*.
(B) A rod of linear mass density λ = |α**x*^3|; that lies on the *x* axis from -*L/2* to +*L/2*.
(C) A cone lies with its axis along the *z* axis and its base centered on the origin of the corrdinate system. The height of the cone is *h* and its base has radius *a*. The density of the cone is ρ = α*z* .
(D) Challenge problem...
A sphere of radius *R* has a charge density ρ = α*x*+β*x*^2; .

2. Find the moments of inertia (*MI* because *I* is reserved in Maple) of the following objects:
(A) A cylinder of uniform density rotated about the axis of the cylinder.
h is the length of the cylinder
R is the radius of the cylinder
m is the mass of the cylinder
r is the cylindrical variable.
(B) A sphere of uniform density rotated about the axis that passes through its center.
R is the radius of the sphere
m is the mass of the sphere
r is the cylindrical variable.
(C) A thin rod of mass *m* and length *L* rotated about a line that is perpendicular to its axis and that passes through its center. Repeat for an axis parallel to this axis, but passing through the end of the rod.
(D) Challenge problem....
Find the moment of inertia of a cylinder rotating about a line is perpendicular to the cylinder axis and passes through the center of the cylinder.
h is the length of the cylinder
R is the radius of the cylinder
m is the mass of the cylinder
r, θ, and *z* are the cylindrical variables with the *z* axis corresponding to the axis of the cylinder.For the following objects, find the total mass and the center of mass.

alpha; and
beta; are constants in all that follow.

(A) A rod of linear mass density
lambda = alpha*x^2; that lies on the x axis from -L/2 to +L/2

(B) A rod of linear mass density
lambda = abs(alpha*x^3); that lies on the x axis from -L/2 to +L/2

(C) A cone lies with its axis along the z axis and its base centered on the origin of the corrdinate system. The height of the cone is h and its base has radius a. The density of the cone is
rho = alpha*z; .

(D) Challenge problem...
A sphere of radius R has a charge density
rho = alpha*x+beta*x^2; .

Problem 6.2

Find the moments of inertia (MI because I is reserved in Maple) of the following objects:

(A) A cylinder of uniform density rotated about the axis of the cylinder.
h is the length of the cylinder
R is the radius of the cylinder
m is the mass of the cylinder
r is the cylindrical variable.

(B) A sphere of uniform density rotated about the axis that passes through its center.
R is the radius of the sphere
m is the mass of the sphere
r is the cylindrical variable.

(C) A thin rod of mass m and length L rotated about a line that is perpendicular to its axis and that passes through its center.
Repeat for an axis parallel to this axis, but passing through the end of the rod.

(D) Challenge problem....
Find the moment of inertia of a cylinder rotating about a line is perpendicular to the cylinder axis and passes through the center of the cylinder.
h is the length of the cylinder
R is the radius of the cylinder
m is the mass of the cylinder
r,
theta;, and z are the cylindrical variables with the z axis corresponding to the axis of the cylinder.