Physics 321
Homework 5

Due at midnight on the day of Hour 7.
Problem 5.3 will not be due this semester!

Collision problems are fairly easy conceptually, but a real pain to actually solve the equations. That's because there are four equations – one energy conservation and three momentum conservation – equations that must be solved simultaneously. The problem is that the equations that relate energy to momentum are quadratic. Even Maple chokes at that.

The equations we need to solve for a two-body final state are usually of the form:

$$K_{tot} = K_1 + K_2 + E_{loss}$$
$$p_{tot} = p_{1z} + p_{2z}$$
$$0 = p_{1y} + p_{2y}$$
$$0 = p_{1x} + p_{2x}$$

Here we have called the direction of the initial momentum the *z* direction for simplicity. Usually we take the scattering plane to be the *x-z* plane, so there are no *y* components, however, this isn't necessary, and cannot be done when there are more than two particles in the final state. The trick we use to reduce this is as follows:

$$p_2^2 = p_{2x}^2 + p_{2y}^2 + p_{2z}^2$$
$$= p_{2x}^2 + p_{2y}^2 + \left(p_{tot} - p_{1z}\right)^2$$
$$= p_{tot}^2 + p_1^2 - 2p_{tot}p_{1z}$$
$$K_2 = \left(K_{tot} - E_{loss}\right) - K_1$$
$$\frac{p_2^2}{2m_2} = \left(K_{tot} - E_{loss}\right) - \frac{p_1^2}{2m_1}$$
$$p_2^2 = 2m_2\left(K_{tot} - E_{loss}\right) - \frac{m_2 p_1^2}{m_1}$$
$$\Rightarrow p_{tot}^2 + p_1^2 - 2p_{tot}p_{1z} = 2m_2\left(K_{tot} - E_{loss}\right) - \frac{m_2 p_1^2}{m_1}$$
$$p_1^2\left(1 + \frac{m_2}{m_1}\right) - p_1 2p_{tot}\cos\theta_1 + p_{tot}^2 - 2m_2\left(K_{tot} - E_{loss}\right) = 0$$

1.An alpha particle is incident on a gold nucleus. The alpha particle raises the gold nucleus to an excited state with excitation energy *Eex*. Call the direction of the alpha's initial momentum the *z* direction. Furthermore, we'll assume that the scattering is in the *x-z* plane. (This can be done without any loss of generality. Why?) The particles may be safely treated without relativistic corrections.

Let *m* be the mass of the alpha particle.
M be the mass of the gold nucleus.
K0 and *p0* be the kinetic energy and momentum of the alpha before the collision.
Ka and *pa* be the kinetic energy and momentum of the alpha after the collision.
θ be the scattering angle of the alpha. (Scattering angles are measured with respect to direction of the incident alpha, the +*z* direction in this case.)
K and *p* be the kinetic energy and momentum of the gold after the collision.
ϕ be the scattering angle of the gold nucleus.

Find *Ka* as a function of θ and the initial parameters, *m*, *M*, *K0*, and *Eex*.

2. Now we will caluclate some numerical values for the scattering desribed on Problem 5.1. The alpha has a kinetic energy of 6.0 MeV (1.0 MeV=1.602e-13 J). The masses of the particles are given in kg in the Maple worksheet.

Make a plot of *Ka* as a function of θ for *Eex*=0 and 0.75 MeV.

3. A rocket can be made from a soda bottle by partially filling it with water and pumping air into it to increase its pressure. The rocket is positioned with its opening to the bottom and the pressure is suddenly released and the rocket flies into the air.

In this problem we model the soda bottle by a cylinder with a hole in the bottom. The rocket is a bit complicated in that the pressure decreases as water leaves, causing the exhaust velocity to vary as a function of height. We can derive a simple relationship by assuming 1) the process is adiabatic (it's fairly fast and somewhat insulated, so it's not a bad approximation) and 2) the water is an ideal fluid. We then apply Bernoulli's Equation and the Continuity Equation to obtain the desired function. These functions are given to you in the Maple worksheet.

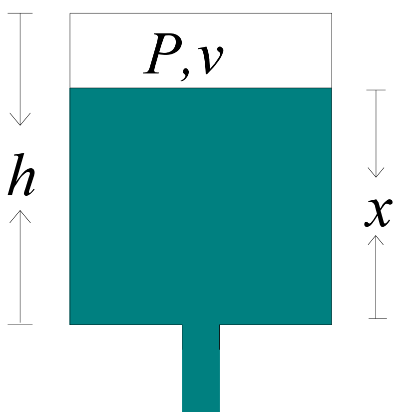
Variables:
A cross-sectional area of the cylinder
a cross-sectional area of the opening
h height of the cylinder
x height of the water
dx_dt rate at which the height of the water in the bottle is changing.
x_i initial height of the water in the bottle
P pressure of the air in the bottle
P_i initial pressure of teh air int eh bottle.
P0 atmospheric pressure
Pb pressure at the bottom of the bottle = $P + \rho gx$.
ρ density of water in kg/m^3;
m mass of the empty bottle
vex exhaust velocity
y height of the rocket
gam the adiabatic factor, γ ("gamma" is reserved, so we can't use it.)

Ignore drag in this calculation.

Notes for the bottle rocket problem:

Consider the expansion of the gas to be adiabatic, as it is effectively insulated from the outside world.

We assume that the initial pressure and volume of the gas, the dimensions of the container, and the adiabatic constant, γ, of the gas are known.



$$P(x)V^{\gamma}(x) = P_i^{\gamma}V_i^{\gamma}$$

$$P(x) = \frac{P_i(h - x_i)^{\gamma}A^{\gamma}}{(h - x)^{\gamma}A^{\gamma}} = \frac{P_i(h - x_i)^{\gamma}}{(h - x)^{\gamma}}$$

$$P(bottom) \equiv P_b = P(x) + \rho g x$$

$$v_{ex} \Rightarrow P_b + 0 + 0 = P_0 + \tfrac{1}{2}\rho v_{ex}^2$$

$$v_{ex} = \sqrt{\frac{2(P_b - P_0)}{\rho}}$$

$$A\left|\frac{dx}{dt}\right| = a v_{ex}$$

$$\frac{dx}{dt} = -\frac{a}{A}v_{ex}$$

$$Thrust = \left|\dot{M}v_{ex}\right|$$