Physics 321
Homework 28

In order to calculate angular momentum for an arbitrary mass rotating about an arbitrary axis, it isn't sufficient to know just a moment of inertia. We need a more complete description of the mass distribution. This is given by the definitions:

$$J_{ij} = \int r_i r_j \rho dV, \quad r_1 = x, r_2 = y, r_3 = z$$
$$I_{11} = J_{22} + J_{33}, \text{ etc.} \Rightarrow \text{ moments of inertia}$$
$$I_{12} = -J_{12}, \text{ etc.} \Rightarrow \text{ products of inertia}$$

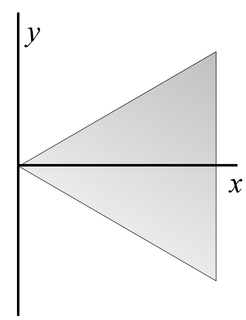
Once we know the moments and products of inertia, we can construnct an inertia tensor. For a given angular velocity, the angular momentum is given by:

$$\vec{L} = \boldsymbol{I}\vec{\omega}$$
$$\begin{pmatrix} L_x \\ L_y \\ L_z \end{pmatrix} = \begin{pmatrix} I_{xx} & I_{xy} & I_{xz} \\ I_{xy} & I_{yy} & I_{yz} \\ I_{xz} & I_{yz} & I_{zz} \end{pmatrix} \begin{pmatrix} \omega_x \\ \omega_y \\ \omega_z \end{pmatrix}$$

Note that, in general, the angular momentum is not parallel to the angular velocity.

Problem 1.

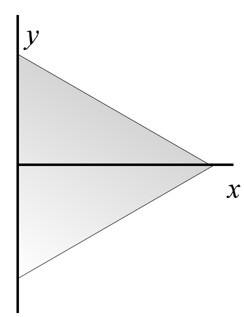
A lamina is in the shape of an equilateral triangle. It has a mass of m = 325 g and is 25.0 cm on a side. (Being a lamina, the thickness is unimportant.) The density of the lamina is uniform.



Find the moments and products of inertia for each of the following cases.

(A) The lamina is in the x-y plane with one one side of the triangle on the y axis and the opposite vertex of the triangle is on the +x axis.

(B) The lamina is in the x-y plane with the vertex ant the origin and the ecnter of one side on the +x axis.



Let
m be the mass of the lamina in kg.
A be the area of the lamina in m^2.
sigma be the density of the lamina in kg/m^2.
L be the length of each side.

Problem 2.

A sphere of radius R has a volume mass density given by relation rho = rho0 + alpha*x*z. Find the mass and the moments and products of inertia of the sphere.