Physics 321
Homework 26

This homework assignment deals with how to modify equations of motion to account for the rotation of the earth. To do this we choose a coordinate system such that *z* is up, *x* is east, and *y* is north. You must rearrange your coordinates to match those directions to use the equations given below.

The equations of motion then become

$$\ddot{x} = F_x + 2\Omega\left(\dot{y}\cos\theta - \dot{z}\sin\theta\right)$$
$$\ddot{y} = F_y - 2\Omega\dot{x}\cos\theta$$
$$\ddot{z} = F_z - g + 2\Omega\dot{x}\sin\theta$$

where $\Omega = 7.27\times 10^{-5}$ rad/s is the angular velocity of the earth.

Note that these equations include the Coriolis force, but not the centrifugal force. This is because we use “Plumb bob” coordinates that already have the centrifugal force built into them.

Problem 1.

Work through the Foucault pendulum problem using Maple.

Use the following information:
Our latitude is very close to 40 degrees north (so what is *theta*?). (Actually 40 degrees passes through Payson.)

L = 6.0 m
Omega is the earth's angular velocity.
At *t* = 0, the pendulum is pulled toward the east to *x0*=1.50 m and then released from rest.
Use the coordinates of the text: *z* is up, *x* is east, and *y* is north.

(A) Review the first two pages of the text and so that you understand the origin of Eq 9.61.
(B) Write down the equations of motion with the appropriate boundary conditions.
(C) Put in numbers and do the solution numerically. Plot *x*(*t*) and *y*(*t*).
(D) Calculate the period of oscillation. If we let *r*=sqrt(*x*^2+*y*^2), then *r*(*t*) should be a simple sine (or cosine, of course) function. For small t, x(t) is essentially the same as *r*(*t*), so see if you can figure out the sine function that represents the solution for *x*(*t*) above. I have provided you with Maple code that should help you compare the curves.
(E) On physical grounds, we expect the *x* value to be decreasing slightly while the *y* value is increasing. (What "physical grounds" require this?)
Also, we expect *y*(*t*) to be approximately the product of a simple sine function (of the same period as *x*(*t*)) and a second sine function of a longer period. This longer period is the time it would take the pendulum to swing along the *x*-axis again. Call this period *T2* and its corresponding angular frequency *Omega2*. By trial and error, fit *Omega2* to *y*(*t*) over the range of 0 to 100 seconds.

Problem 2

A large artillery shell has a range of 2.200 km when fired at an angle of 40 degrees. Let *z* be up and *x* be the direction the shell is fired.

Let
m = 100 kg be the mass of the shell.
d = 0.35 m be the diameter of the shell.
Omega is the angular velocity of the earth.
phi = 50 degrees is the colatitude (90 degrees - latitude).

(A) Find the muzzle velocity of the shell. Include quadratic drag, assuming that the formula for a sphere apples here.
(B) Now use Equations 9.53 with drag added in to find how much the rotation of the earth affects the location where the the shell strikes the ground. Let the shell be fired due east. (Note that east is the x direction in equation 9.53.)
(C) Based on your graphs, find *Delta x* and *Delta y*, the difference in position caused by the rotation of the earth. Put your answers in meters.