Physics 321
Homework 25

The following relate velocities and accelerations in inertial (S_0) and noninertial (S) frames:

$$\begin{pmatrix}\hat{x}_0 & \hat{y}_0 & \hat{z}_0\end{pmatrix}\begin{pmatrix}\dot{x}_0\\ \dot{y}_0\\ \dot{z}_0\end{pmatrix} = \begin{pmatrix}\hat{x} & \hat{y} & \hat{z}\end{pmatrix}\left[\begin{pmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{pmatrix} + \vec{\Omega}\times\begin{pmatrix}x\\ y\\ z\end{pmatrix}\right] \quad\Rightarrow\quad \vec{v}_0 = \vec{v} + \vec{\Omega}\times\vec{r}$$

$$\begin{pmatrix}\hat{x} & \hat{y} & \hat{z}\end{pmatrix}\begin{pmatrix}\dot{x}\\ \dot{y}\\ \dot{z}\end{pmatrix} = \begin{pmatrix}\hat{x}_0 & \hat{y}_0 & \hat{z}_0\end{pmatrix}\left[\begin{pmatrix}\dot{x}_0\\ \dot{y}_0\\ \dot{z}_0\end{pmatrix} - \vec{\Omega}_0\times\begin{pmatrix}x_0\\ y_0\\ z_0\end{pmatrix}\right] \quad\Rightarrow\quad \vec{v} = \vec{v}_0 - \vec{\Omega}_0\times\vec{r}_0$$

$$\ddot{\vec{r}}_0 = \ddot{\vec{r}} + \dot{\vec{\Omega}}\times\vec{r} + 2\vec{\Omega}\times\dot{\vec{r}} + \vec{\Omega}\times\left(\vec{\Omega}\times\vec{r}\right) + \vec{A}$$

$$\ddot{\vec{r}} = \ddot{\vec{r}}_0 - \dot{\vec{\Omega}}_0\times\vec{r}_0 - 2\vec{\Omega}_0\times\dot{\vec{r}}_0 + \vec{\Omega}_0\times\left(\vec{\Omega}_0\times\vec{r}_0\right) - \vec{A}$$

$$m\ddot{\vec{r}} = \vec{F}_{real} + m\vec{r}\times\dot{\vec{\Omega}} + 2m\dot{\vec{r}}\times\vec{\Omega} + m\left(\vec{\Omega}\times\vec{r}\right)\times\vec{\Omega}$$

You do not need to memorize these equations, but you should be able to use them, explain the meaning of each term, and give examples of problems in which they play a role.

You should also be able to rotate from S to S_0 and back. The following is a summary of how rotations work for a turntable with angular velocity Ω in the *z* direcetion:

$$\begin{pmatrix}\hat{x} & \hat{y} & \hat{z}\end{pmatrix} = \begin{pmatrix}\hat{x}_0 & \hat{y}_0 & \hat{z}_0\end{pmatrix}R$$

$$\begin{pmatrix}\hat{x}_0 & \hat{y}_0 & \hat{z}_0\end{pmatrix} = \begin{pmatrix}\hat{x} & \hat{y} & \hat{z}\end{pmatrix}R^{-1}$$

$$\begin{pmatrix}x\\ y\\ z\end{pmatrix} = R^{-1}\begin{pmatrix}x_0\\ y_0\\ z_0\end{pmatrix} \qquad \begin{pmatrix}x_0\\ y_0\\ z_0\end{pmatrix} = R\begin{pmatrix}x\\ y\\ z\end{pmatrix}$$

$$R = \begin{pmatrix}\cos\Omega t & -\sin\Omega t & 0\\ \sin\Omega t & \cos\Omega t & 0\\ 0 & 0 & 1\end{pmatrix}$$

Again, you don't need to memorize these, but be sure you understand how to use them.

Problem 1.

A turntable spins with angular velocity Omega in the +z direction. A bee flies past at position given by (*a*t^2/2, y0, 0*) for *t* > 0. Find the position, velocity, and acceleration of the bee as a function of time, as measured by an observer on the turntable.

(A) Do the problem using a rotation matrix and derivatives.
(B) Do the problem using the Eq. 9.34 or generalizations of it.
(C) Graph the bee's trajectory and velocity.

Hint: You may wish to refer to the Maple Exercises online. -- Be sure you understand what you're doing, however!

Let
a = 5.00 m/s^2
$y0$ = 1.0 cm

Let *Ovec* be the angular velocity vector.

Problem 2.

A turntable has angular velocity *Omega* (in the z direction) and angular accelration *Alpha* > 0 (also in the z direction). A tube extends from the center of the turntable to its edge. A spring connects a rod on the turntable's axis to a mass inside the the tube. The mass slides without friction in the tube. The spring constant is k, the mass is m, and the equilibrium length of the spring is *L0*. Let x be the distance of the mass from the center rod.

Find the position of the mass as a function of time and the force of the tube on the mass (call this N) as a function of time.

Let
m = 15.0 g
Omega = 30 rad/s
Alpha = 5 rad/s^2
L0 = 40.0 cm
k = 16 N/m^2

Let *Ovec* be the angular velocity vector and *Avec* be the angular acceleration vector.

Do the problem using Eq. 9.34, generalizing it to include the transverse force caused by the angular acceleration.