

Physics 321
Homework 20

Due at midnight on the day of Hour 21.

The keys to working with the Hamiltonian are:

1) Find the Lagrangian.

2) Find the conjugate momenta from the Lagrangian: $p_i = \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$.

3) Solve for \dot{q}_i in terms of p_i and q_i .

4) Use this to express T in terms of the conjugate momenta.

5) $H = T + U$ for all the problems we'll do.

6) Use the equations: $\frac{\partial H}{\partial q_i} = -\dot{p}_i$, $\frac{\partial H}{\partial p_i} = \dot{q}_i$

Problems:

1. Two point masses each of mass m are joined by a spring of spring constant k . The distance between each mass and the center of mass is z . The of equilibrium length of the spring is $2z_0$. The masses slide without friction on an air table.

x, y are the coordinates of center of mass

θ angular coordinate for rotational motion of the masses

2. This is an online example, but you'll rework it with different coordinates.

A mass m_1 is connected by a spring to a wall. A mass m_2 is connected to mass m_1 by an identical spring. We let x_1 be the distance of m_1 from the wall, and x_2 be the distance of m_2 from the wall. The springs are identical with spring constant is k and the equilibrium length of the spring is x_0 .

Compress each spring to a length $x_0/2$ and then release them from rest. Plot $x_1(t)$, $x_2(t)-x_1(t)$ and $x_2(t)$.

$$m_1 = 0.250$$

$$m_2 = 0.125$$

$$x_0 = 0.3$$

$$k = 1.24$$

(You may ignore the length of each mass.)