

Physics 321
Homework 18

Due at midnight on the day of Hour 19.

- We can find forces of constraint using Lagrange multipliers.

Write the equation of constraints in variables (we'll call them x and y) as

$$f(x,y) = \dots = 0$$

Then, Lagrange's Equations with forces of constraint become:

$$F_x = \lambda \frac{\partial f}{\partial x} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{x}} - \frac{\partial \mathcal{L}}{\partial x}$$
$$F_y = \lambda \frac{\partial f}{\partial y} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{y}} - \frac{\partial \mathcal{L}}{\partial y}$$

Method of solution:

- 1) Find T and U in terms of all the variables (ignore constraints).
- 2) Write the equations of motion.
- 3) Solve for λ in one equation and put this value in the other.
- 4) Use the constraint relation to eliminate one variable.
- 5) Solve the equation of motion.
- 6) Solve for the forces of constraint in terms of the solution to equation of motion.

Maple notes:

- 1) A lot of this has to be done by hand.
- 2) When the solution is obtained numerically, you have to do some work to find the forces of constraint, as Maple won't plot second derivatives in ODEPLOT.

Problems.

1. A small block slides frictionlessly in a parabolic bowl. Assume the object is brought to a certain height and released from rest, so that the motion can be confined to two dimensions. Find the motion of the block using Lagrange's Equations with Lagrange multipliers.

The equation of the bowl is $y = \alpha x^2$

$m = 100\text{g}$

$\alpha = 6.00$

The initial value of x is $x_0 = 30\text{ cm}$.

2. Continue the problem to find the forces of constraint.

Be sure you understand what the solution is telling you!