

Physics 321
Homework 16

Due at midnight on the day of Hour 17.

- Lagrange's Equations for simple systems form a "turn the crank" approach to problems.

- 1) Determine the number of degrees of freedom and the natural variables, q_i .
- 2) Write down T and U in terms of q_i and \dot{q}_i .

Maple hint: Call these qi and qi_dot . Do not make the time dependence explicit at this point.

- 3) Form $L = T - U$. Take derivatives with respect to q_i and \dot{q}_i .
- 4) Construct one equation of motion by using the formula

$$\frac{\partial \mathcal{L}}{\partial q_i} = \frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{q}_i}$$

for each variable in turn.

Maple hint: Cut and paste the derivatives into the equation of motion, changing each qi into $q(t)$ and each qi_dot into $\text{diff}(qi(t),t)$ in the process.

- Although the answers come out easily, it is critically important to understand what the solutions are telling you.

Problems

1. Write down the Lagrangian for a simple function as a function of θ and $\dot{\theta}$.

m is the mass of the pendulum (the rod is massless).

d is the length of the pendulum.

θ is the pendulum angle. It is 0 when the pendulum is straight down.

$\dot{\theta}$ is $\dot{\theta}$.

Find the equation of motion for the system.

Find the equation of motion for the small angle approximation.

Find $\theta(t)$ for the pendulum for initial conditions of $\theta = \pi/6$ and $\dot{\theta} = 0.1$.

For the solutions, let $d = 0.2482$ m and $m = 0.5$ kg.

2. A small mass is placed near the top of a smooth sphere and released from rest. Find the motion of the mass as it slides down the sphere. Assume that the system is frictionless.

m is the mass of the block.

R is the radius of the sphere

θ is the usual spherical polar angle with $\theta = 0$ going up.

Let the mass be 100 g, the radius of the sphere be 0.45 m, and the initial angle be $\pi/12$.

3. A mass m moves in a three-dimensional, nonlinear harmonic oscillator of potential $U = 1/2 * k r^2 + 1/4 * s r^4$. Also include gravitational potential energy. Work in Cartesian coordinates and let the upward direction be z .

The values of the constants are $k = 0.3$, $s = 100$ in SI units.

The initial conditions are: $x(0) = 0.0005$ m, $v_x(0) = 0$, $y(0) = 0$, $v_y(0) = 0.1$ m/s, $z(0) = 0$, $v_z(0) = -0.03$ m/s.