Physics 321
Homework 15

Due at midnight on the day of Hour 16.

Lagrange's Equations of Motion are the consequence of a variational principle: Nature chooses a process that makes the integral of the Lagrangian, $L = T - U$, over time to be an extremum of the motion.

For this assignment, I simply want you to become familiar with applying the Euler-Lagrange Equation, Eq. 6.13, to another application. I have chosen as the application two problems involving Fermat's Principle.

Problems:

1. A ray of light is incident at an angle of 60 degrees on a surface of water. Use Fermat's principle (Eq. 6.3) and the Euler-Lagrange Equation (Eq. 6.13) to find the path of the light as it passes into the surface of the water.

Since discontinuous functions present some problems, consider the index of refraction to be of the form: n:=1.33-.33/(1+exp(-y/0.005)).

(Note that if the we reduce the value of the constant in the exponent to 0.001, the change in slope is so large that the numerical solution to the differetial equation is no longer valid.)

2. Near a hot road, the index of refraction is given by the function: *n(y)=1+alpha*y* where *alpha* is a constant that will can be adjusted later. Follow the example of Problem 15.1 to find the path of light initially at *y*=20 m and traveling at a slope of -0.4.

(Note, the speed of light is faster near the ground.)