Physics 321
Homework 12

Due at midnight on the day of Hour 13.

Driven harmonic oscillators:

$$\ddot{x} + 2\beta\dot{x} + \omega_0^2 x = f(t) = F(t)/m$$

For sinusoidally driven oscillators:

Solutions are solutions to the damped, undriven oscillaor + a solution to the driven oscillator.

The particular solution to the driven oscillator is the steady-state solution, the solution to the damped osicllator is the transient solution.

We usually take a complex solution and keep the real part.

The basic solution is

$$x(t) = A\cos(\omega t - \delta)$$ (memorize)

where

$$A^2 = \frac{f_0^2}{\left(\omega_0^2 - \omega^2\right)^2 + 4\beta^2\omega^2}$$

$$\tan\delta = \frac{2\beta\omega}{\omega_0^2 - \omega^2}$$ (don't memorize)

Be able to sketch $A^2(\omega)$ and $\delta(\omega)$.

The width of the A^2 curve is approximately 2β.

Q (quality) factor

$$Q = \frac{\omega_0}{2\beta} = \pi\frac{decay\ time}{period}$$

Problem:

1. A 5.00 g mass made of iron hangs vertically from a spring of spring constant 6.00 N/m. It is driven sinusoidally by an electromagnet that provides a maximum force 0.0005 N on the mass.The system is characterized by a damping constant of beta = 2.

(A) Find the differential equation for the system with the driving frequency omega left as a variable. Plot the early and late behavior of the system for three driving frequencies: 2 omega0, 1/2 omega0 and omega0. Take the initial conditions to be x(0)=0 m and v(0)=0.

B) From the solution of the differential equation above, find the part that corresponds to the steady state solution and cut and paste it into the finction below. Plot this function and find the maxium amplitude of the solution from the equation.

(C) Using the equation for A derived in class or the text (Eq. 5.64), plot A as a function of the driving frequency. Call the driving frequency omegad, as we have a fixed value for omega already defined. Be sure that the values of A you obtained for the three frequencies above are consistent with this curve

(D) Now plot A^2 as a function of the driving frequency. Estimate the FWHM of this curve graphically. (Note this can be done most easily by plotting the curve for values of A^2 ranging from A0^2/2 to A0^2. Compare this number with 2*beta. Determine the quality factor, Q, of the oscillator.