

Physics 321
Homework 11

Due at midnight on the day of Hour 12.

Undamped harmonic oscillators:

$$F = m\ddot{x} = -kx$$

$$\ddot{x} + \omega^2 x = 0, \quad \omega_0 = \sqrt{k/m}$$

$$x(t) = A \cos(\omega t - \delta)$$

Damped harmonic oscillators:

$$F = m\ddot{x} = -kx - c\dot{x}$$

$$\ddot{x} + 2\beta\dot{x} + \omega^2 x = 0$$

Underdamped $\beta < \omega_0$

$$x(t) = Ae^{-\beta t} \cos(\omega_1 t - \delta), \quad \omega_1 = \sqrt{\omega_0^2 - \beta^2}$$

Critically damped $\beta = \omega_0$

$$x(t) = Ae^{-\beta t} + Bte^{-\beta t}$$

Overdamped $\beta > \omega_0$

$$x(t) = Ae^{-(\beta - \sqrt{\beta^2 - \omega_0^2})t} + Be^{-(\beta + \sqrt{\beta^2 - \omega_0^2})t}$$

Problems

1. A small mass of $m = 15.0$ g slides frictionlessly in a bowl that has a height given by the function $z(x, y) = 0.1x^2 + Cy^2$ where all the constants are in SI units. Assume that the z component of velocity is small enough that it can be ignored.

(A) For exercise, write down U as a function of x and y , and take the gradient of it to find the force. Then solve differential equations in x and y to find the frequency. You need not include initial conditions.

Find the value of C that gives a frequency 3 times larger in the y equation than in the x equation.

(B) Make a graph of the motion for initial conditions $x(0)=0.2$ m, $y(0)=0$, $v_x(0)=0$, $v_y(0)=0.5$ m/s.

2. Play around with the damped HO equation with differing values of the damping constant. Make plots for underdamped, overdamped, and critically damped cases.

3. A massless spring of spring constant $k=3.24 \text{ N/m}$ is hung from the ceiling. We call the bottom end of the spring $y=0$. A mass of 125 g is hung from the spring. It takes 24.0 sec for the amplitude of the spring's oscillations to reduce to $1/2$ of their initial value. What is the value of the damping constant? What is the value of β/ω_0 ? What is the oscillation frequency, ω_1 ?