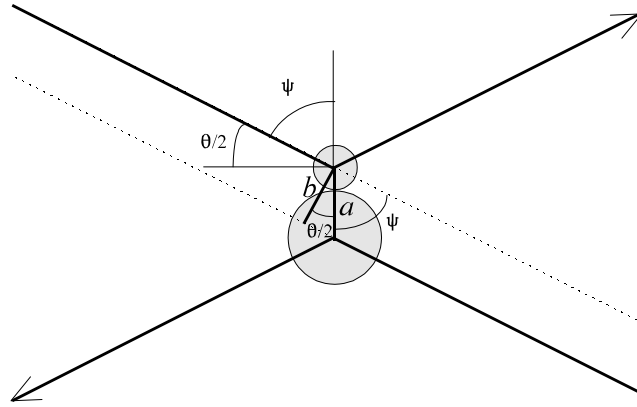


Impact Parameter and Scattering Angle

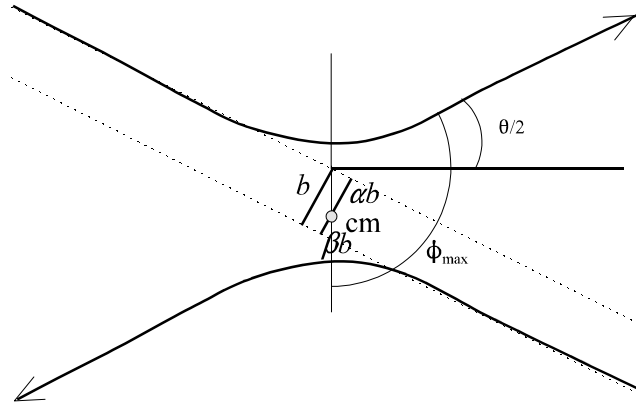
Hard sphere scattering



$$\cos\left(\frac{\theta}{2}\right) = \sin\left(\frac{\pi - \theta}{2}\right) = \frac{b}{a}$$

$$b = a \sin\left(\frac{\pi - \theta}{2}\right)$$

Coulomb (Rutherford) scattering



$$\ell = p\alpha b + p\beta b = pb, \quad T_0 = T(r \rightarrow \infty) = \frac{p^2}{2\mu} = \frac{\ell^2}{2\mu b^2} = E$$

$$E = \frac{\gamma^2 \mu}{2\ell^2} (\epsilon^2 - 1), \quad \gamma = kq_1 q_2$$

$$r(\phi) = \frac{C}{1 + \epsilon \cos \phi}, \quad \cos \phi_{\max} = -\frac{1}{\epsilon}, \quad \phi_{\max} = \frac{\pi}{2} + \frac{\theta}{2}$$

$$\cos \phi_{\max} = \cos\left(\frac{\pi}{2} + \frac{\theta}{2}\right) = -\sin \frac{\theta}{2} = -\frac{1}{\epsilon}, \quad \cos \frac{\theta}{2} = \sqrt{1 - \frac{1}{\epsilon^2}} = \frac{\sqrt{\epsilon^2 - 1}}{\epsilon}$$

$$\cot^2 \frac{\theta}{2} = \epsilon^2 - 1 = \frac{2\ell^2 E}{\gamma^2 \mu} = \frac{2(2\mu b^2 E)E}{\gamma^2 \mu} = \frac{4b^2 E^2}{\gamma^2}$$

$$\cot \frac{\theta}{2} = \frac{2bT_0}{\gamma}$$