1. A toy gyroscope is set in motion and one axis is placed on a stand as illustrated in the diagram. The gravitational torque causes the gyroscope to precess in a clockwise direction when viewed from the top. What is the direction of the angular momentum. Explain how you deduced this fact.

*The gravitational force would cause a torque into the page. This is the direction of the change in the angular momentum vector. That is consistent with the angular momentum pointing toward the pivot point (to the left).*

2. A mass $m$ is attached to a horizontal spring that has a spring constant $k$. When the spring is not in motion, it is located at a position $x_0$. The string is compressed to a position $x_f$ and then released from rest. Using conservation of energy in two different ways, write down equations of motion for $x(t)$. (I want your equations to be in terms of $x$ and its derivatives, not $v, a, etc$.) Also write down any initial conditions that must be included to solve the equations.

\[(A \text{ or } B) \quad T + U = E \]
\[
\frac{1}{2} m \dot{x}^2 + \frac{1}{2} k (x - x_0)^2 = E, \quad E = \frac{1}{2} k (x_1 - x_0)^2, \quad x(0) = x_1
\]

\[(B \text{ or } A) \quad \ddot{T} + \ddot{U} = 0 \]
\[
m \dddot{x} + k (x - x_0) \ddot{x} = 0 \]
\[
\Rightarrow \ddot{x} = 0, \quad \text{or} \quad m \ddot{x} + k (x - x_0) = 0, \quad x(0) = x_1, \quad \dot{x}(0) = 0
\]
3. Derive expressions for the cyclotron radius and the cyclotron frequency. (The radius and frequency for charged particles that move in circular orbits in a constant $B$ field.)

\[ \frac{mv^2}{r} = qvB \]

\[ p = qBr, \quad r = \frac{p}{qB} \]

\[ \frac{v}{r} = \frac{qB}{m}, \quad \omega = \frac{qB}{m}, \quad \text{or} \quad \frac{f}{2\pi} = \frac{qB}{2\pi m} \]

4. A mass is connected to a vertical spring. Differently shaped objects can be attached to the mast to adjust the damping constant of the system.

(A) The spring is stretched to a position of +20 cm. On the graph to the right, sketch the approximate displacement of the spring as a function of time for critical damping, for overdamping, and for underdamping. The last two cases, assume that the damping is very nearly critical. (Of course, the time constant is arbitrary since I haven’t given you detailed numbers.)

*Roughly, as to the right. The time scale isn’t significant.*

(B) How does $\beta$ compare to $\omega_0$ for each of the three cases?

*Overdamping:* $\beta > \omega_0$

*Critical damping:* $\beta = \omega_0$

*Underdamping:* $\beta < \omega_0$
5. Below is a sketch of $A^2$ for a harmonic oscillator as a function of the driving frequency $\omega$. What is the damping constant $\beta$? What is $Q$ for the oscillator?

$\beta = 20 \text{ sec}^{-1}$
$Q = 25$

To deduce these, you need to know:

FWHM is the full width of a peak measured at half its maximum value.

$\text{FWHM} = 2\beta$

$Q = \frac{\alpha_0}{\text{FWHM}}$

6. A baseball flies in the $x$, $y$, and $z$ directions. Write down the equations of motion with linear and quadratic drag. (You don’t need to include the equations with diameter in them, and you don’t need to include wind.)

$m\ddot{x} = -b\dot{x} - cv\dot{x}$
$m\ddot{y} = -b\dot{y} - cv\dot{y}$
$m\ddot{z} = -b\dot{z} - cv\dot{z} - mg$

$v = \sqrt{x^2 + y^2 + z^2}$

or something equivalent.

7. Show how the Lorentz force can violate Newton’s Third Law of Motion.

Take two charges moving at right angles to each other. Find the field on one charge from the other charge (use the right-hand rule as for a current-carrying wire). Find the magnetic part of the force by

$\vec{F}_B = q\vec{v} \times \vec{B}$. 