Exercise 8.3.

4. We use $H$ for the Heaviside Function, as we’ve run out of $U$s.

\[ u_t = u_{xx}, \quad u(0,t) = 100[H(t-1) - H(t-3)], \quad u(x,0) = 0 \]

\[ sU(x,s) = U_{xx}(x,s), \]

\[ U(x,s) = A(s)e^{\sqrt{s}x} \]

\[ U(0,s) = A(s) = L(u(0,t)) = \frac{100}{s} [e^{-s} - e^{-3s}] \]

\[ U(x,s) = L\left(100[H(t-1) - H(t-3)]\right)L\left(-\frac{x e^{-x^2/(4t)}}{2\sqrt{\pi} t^{3/2}}\right) \]

The inverse transform is done with Maple.

\[ u(x,t) = -\frac{50x}{\sqrt{\pi}} \int_0^t [H(t - \tau - 1) - H(t - \tau - 3)] \frac{e^{-x^2/(4\tau)}}{\tau^{3/2}} d\tau \]

Also see the Maple worksheet.

8.

\[ u_t = u_{xx} + t^2, \quad u(0,t) = 0, \quad u(x,0) = 0, \quad u_t(x,0) = 0 \]

\[ s^2U(x,s) = U_{xx}(x,s) + \frac{2}{s^3} \]

\[ U(x,s) = A(s)e^{-sx} + B(s)e^{sx} + \frac{2}{s^3} \]

\[ B(s) = 0 \text{ so the solution will be finite at large } x \]

\[ U(0,s) = A(s) + \frac{2}{s^3} = 0 \]

\[ U(x,s) = \frac{2}{s^3} [1 - e^{-sx}] \]

\[ u(x,t) = H(t-x)\left[-\frac{1}{12}t^4 + \frac{1}{3}t^3x - \frac{1}{2}t^2x^2 + \frac{1}{3}tx^3 - \frac{1}{12}x^4\right] + \frac{1}{12}t^4 \]

Also see the Maple worksheet.

See Maple worksheets for 11 and 14.