Exercise 8.3.

4. We use $H$ for the Heaviside Function, as we've run out of $U$s.

$$u_t = u_{xx}, \quad u(0,t) = 100\left[H(t-1) - H(t-3)\right], \quad u(x,0) = 0$$
$$sU(x,s) = U_{xx}(x,s),$$
$$U(x,s) = A(s)e^{\sqrt{s}x}$$
$$U(0,s) = A(s) = L(u(0,t)) = \frac{100}{s}\left[e^{-s} - e^{-3s}\right]$$
$$U(x,s) = L\left(100\left[H(t-1) - H(t-3)\right]\right)L\left(-\frac{xe^{-x^2/(4t)}}{2\sqrt{\pi}t^{3/2}}\right)$$

The inverse transform is done with Maple.

$$u(x,t) = -\frac{50x}{\sqrt{\pi}}\int_0^t \left[H(t-\tau-1) - H(t-\tau-3)\right]\frac{e^{-x^2/(4\tau)}}{\tau^{3/2}}\,d\tau$$

Also see the Maple worksheet.

8.

$$u_{tt} = u_{xx} + t^2, \quad u(0,t) = 0, \quad u(x,0) = 0, \quad u_t(x,0) = 0$$
$$s^2U(x,s) = U_{xx}(x,s) + \frac{2}{s^3},$$
$$U(x,s) = A(s)e^{-sx} + B(s)e^{sx} + \frac{2}{s^5}$$

$B(s) = 0$ so the solution will be finite at large $x$

$$U(0,s) = A(s) + \frac{2}{s^5} = 0$$
$$U(x,s) = \frac{2}{s^5}\left[1 - e^{-sx}\right]$$
$$u(x,t) = H(t-x)\left[-\frac{1}{12}t^4 + \frac{1}{3}t^3x - \frac{1}{2}t^2x^2 + \frac{1}{3}tx^3 - \frac{1}{12}x^4\right] + \frac{1}{12}t^4$$

Also see the Maple worksheet.

See Maple worksheets for 11 and 14.