Exercise 7.3.

23. First we take the Fourier transform of the equation:

\[ u_t = c^2 u_{xx} + ku_x \]
\[ \frac{d}{dt} \hat{u}(\omega, t) = -\omega^2 c^2 \hat{u}(\omega, t) + i\omega k \hat{u}(\omega, t) \]

We then solve this equation with Maple:

\[ \hat{u}(\omega, t) = A(\omega) e^{-(\omega^2 c^2 - i\omega k)t} \]
\[ \hat{u}(\omega, 0) = A(\omega) = \hat{f}(\omega) \]
\[ \hat{u}(\omega, t) = \hat{f}(\omega) e^{-(\omega^2 c^2 - i\omega k)t} \]

24. See Maple worksheet. Note why it is that this equation represents convection in the bar.

25. Again, we take the Fourier transform of the equation:

\[ u_t = c^2 u_{xxx} \]
\[ \frac{d}{dt} \hat{u}(\omega, t) = -\omega^3 c^2 \hat{u}(\omega, t) \]

This clearly has solutions:

\[ \hat{u}(\omega, t) = A(\omega) e^{-\omega^3 c t} \]
\[ \hat{u}(\omega, 0) = A(\omega) = \hat{f}(\omega) \]
\[ u(x, t) = \mathcal{F}^{-1}(\hat{f}(\omega) e^{-\omega^3 c t}) \]