Exercise 5.3

1(d). Just plug in the values....

10. Since the origin is included in the region of interest, our solutions must be of the form:

\[
    u(r, \theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} A_{lm} \left( \frac{r}{a} \right)^l Y_{lm}(\theta, \phi)
\]

\[
    u(a, \theta, \phi) = \sum_{l=0}^{\infty} \sum_{m=-l}^{l} A_{lm} Y_{lm}(\theta, \phi) = f(\theta, \phi)
\]

\[
    A_{lm} = \int_{0}^{2\pi} \int_{0}^{\pi} f(\theta, \phi) \overline{Y_{lm}(\theta, \phi)} \sin \theta d\theta d\phi
\]

This is easily evaluated, as \( f \) is given in terms of spherical harmonics.

\[
    A_{lm} = \int_{0}^{2\pi} \int_{0}^{\pi} \left( Y_{10}(\theta, \phi) + 3Y_{11}(\theta, \phi) \right) \overline{Y_{lm}(\theta, \phi)} \sin \theta d\theta d\phi
\]

\[
    A_{10} = 1, \quad A_{11} = 3, \quad \text{all others are zero.}
\]

\[
    u(r, \theta, \phi) = A_{10} \left( \frac{r}{a} \right)^1 Y_{10}(\theta, \phi) + A_{11} \left( \frac{r}{a} \right)^1 Y_{11}(\theta, \phi)
\]

\[
    = \frac{3}{\sqrt{\pi}} \frac{r}{2a} \cos \theta - \frac{3}{\sqrt{2\pi}} \frac{3r}{2a} \sin \theta e^{i\phi}
\]

12. See the Maple Solution.