Exercise 3.2

6. (a) Take a small section of a freely hanging chain. The forces are a tension up, a tension down, and the weight of the segment. Since the chain is at rest, the acceleration is zero. We choose y=0 to be at the bottom of the chain. We then have:

\[(T + \Delta T) - T - W = 0\]
\[\Delta T = W = g\rho \Delta y\]
\[\frac{\Delta T}{\Delta y} = \rho g\]
\[T = \rho gy + \text{Const} = \rho gy\]

We know the constant in the last step is zero because the tension at y=0 is zero. (Why?)

(b–c) We now take the same segment, but allowing a small displacement in the x direction. In this case, the tension will remain essentially the same as in part(a) and \(\theta\) can be treated as a small angle (\(\sin \theta = \theta, \cos \theta = 1\)). We have:

\[(T + \Delta T) \sin(\theta + \Delta \theta) - T \sin \theta = \rho \Delta y x_n\]
\[(T + \Delta T) (\theta + \Delta \theta) - T\theta = \rho \Delta y x_n\]
\[T\theta + T\Delta \theta + \theta \Delta T - T\theta + \Delta T \theta = \rho \Delta y x_n\]
\[T\Delta \theta + \theta \Delta T = \rho \Delta y x_n\]
\[\rho g y \Delta \theta + \theta \rho g \Delta y = \rho \Delta y x_n\]
\[\rho g y \frac{\Delta x}{\Delta y} + \frac{\Delta x}{\Delta y} \rho g \Delta y = \rho \Delta y x_n\]
\[gy \frac{\Delta x}{\Delta y} + \frac{\Delta x}{\Delta y} g = x_n\]
\[gy x_{yy} + gx_y = x_n\]

Note that I have used different notation and different variables than the book. I’ll expect you to use the book’s notation. The values of \(T\) and \(\Delta T\) are obtained from part(a). Remember that the slope, \(\Delta x / \Delta y\), is \(\tan \theta\) and this is approximately equal to \(\theta\) itself.
7. (a) The “−” on the expression for \( F(x,t) \) represents direction. Let’s just look at the direction of the forces and ignore the minus sign. If we think of three balls connected by springs, with the springs stretched, then the force on the middle ball from left ball will be to the left, and the force on the middle ball from the right ball will be to the right. Therefore:

\[
ma = F(x + \Delta x) - F(x) \\
\rho A \Delta x u_n = A E u_x(x + \Delta x) - A E u_x(x)
\]

(b) We just continue by dividing by \( A \Delta x \):

\[
\rho u_n = \lim_{\Delta x \to 0} \frac{E u_x(x + \Delta x) - E u_x(x)}{\Delta x} \\
\rho u_n = E u_{xx} \\
u_n = \frac{E}{\rho} u_{xx} = c^2 u_{xx}
\]