Exercise 2.2

5–16. See the Maple worksheets. Be sure to see how different numbers of terms in the partial sums affect the outcome. Note over what range the Fourier series matches the original function. Note the Gibbs phenomenon at the discontinuities. See how summing over many terms helps reduce the overshoot at the discontinuities.

17.

\[ x^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos nx \]

\[ \pi^2 = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^n}{n^2} \cos n\pi = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{(-1)^{2n}}{n^2} = \frac{\pi^2}{3} + 4 \sum_{n=1}^{\infty} \frac{1}{n^2} \]

\[ \sum_{n=1}^{\infty} \frac{1}{n^2} = \frac{1}{4} \left( \pi^2 - \frac{\pi^2}{3} \right) = \frac{\pi^2}{6} \]

25. See the Maple worksheet.