Chapter 1 – A Tutorial on Linear Algebra

Before we begin our discussion of relativity, it will be useful to introduce a few concepts of linear algebra. You may not have studied matrix operations before; however, if you learn a few basics now, it will make our discussion of relativity considerably easier.

1–1. Systems of Linear Equations

A vector is a collection of related numbers. We usually talk about vectors in three-dimensional space and think of the vectors as having three Cartesian components; however, a vector is much more general than this. Because we are familiar with position vectors, let us begin there. We can define a vector $\boldsymbol{r}$ by the usual expression:

$$\boldsymbol{r} = x\hat{x} + y\hat{y} + z\hat{z}. \tag{1-1}$$

This vector can also be written in "column vector" notation as:

$$\boldsymbol{r} = \begin{pmatrix} x \\ y \\ z \end{pmatrix}. \tag{1-2}$$

We could also write this as a row vector. We call the row vector the "transpose" of the column vector, and use the superscript T to denote the transpose. Thus:

$$\boldsymbol{r}^{\boldsymbol{T}} = \begin{pmatrix} x & y & z \end{pmatrix}. \tag{1-3}$$

Vector operations such as the dot product can be written in this notation as well:

$$\boldsymbol{r_1} \cdot \boldsymbol{r_2} = \boldsymbol{r_1^T}\boldsymbol{r_2} = \begin{pmatrix} x_1 & y_1 & z_1 \end{pmatrix} \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix} = x_1 x_2 + y_1 y_2 + z_1 z_2. \tag{1-4}$$

Note that when we take the dot product of two vectors, we always multiply a row vector by a column vector. This may seem like a needless formalism, but it is motivated by the way we multiply matrices, as we shall see later.

Now let's assume we have a set of linear equations in three variables, x, y, and z. These could be abstract variables, but for sake of concreteness, we consider them to be the ususal Cartesian coordinates in three-dimensional space. We write these equations as:

$$\begin{aligned} A_{xx}x + A_{xy}y + A_{xz}z &= b_x \\ A_{yx}x + A_{yy}y + A_{yz}z &= b_y \\ A_{zx}x + A_{zy}y + A_{zz}z &= b_z \end{aligned} \tag{1-5}$$

Note that the A_{ij} are constants. We may write this same information in matrix notation as:

$$\begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} \tag{1-6}$$

We call the collection of the nine A coefficients a “matrix,” $\boldsymbol{A}$. Then in shorthand notation, we may express this as

$$\boldsymbol{A}\,\boldsymbol{r} = \boldsymbol{b}. \tag{1-7}$$

Although the form looks like simple multiplication, we must remember that, by definition, (1-7) is identical to (1-5). Note that we can also think of $\boldsymbol{A}$ as either a column vector of three row vectors or as a row vector of three column vectors.

$$\begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix} = \begin{pmatrix} \boldsymbol{A^T}_x \\ \boldsymbol{A^T}_y \\ \boldsymbol{A^T}_z \end{pmatrix} = \begin{pmatrix} \boldsymbol{a_x} & \boldsymbol{a_y} & \boldsymbol{a_z} \end{pmatrix} \tag{1-8}$$

where the notation used for the row and column vectors should be self-evident, though perhaps cumbersome. In this notation, the multiplication of a column vector by a matrix can be written as a set of three dot products:

$$\begin{pmatrix} A_{xx} & A_{xy} & A_{xz} \\ A_{yx} & A_{yy} & A_{yz} \\ A_{zx} & A_{zy} & A_{zz} \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} \boldsymbol{A^T}_x \\ \boldsymbol{A^T}_y \\ \boldsymbol{A^T}_z \end{pmatrix}\boldsymbol{r} = \begin{pmatrix} \boldsymbol{A^T}_x\boldsymbol{r} \\ \boldsymbol{A^T}_y\boldsymbol{r} \\ \boldsymbol{A^T}_z\boldsymbol{r} \end{pmatrix} = \begin{pmatrix} \boldsymbol{A}_x\cdot\boldsymbol{r} \\ \boldsymbol{A}_y\cdot\boldsymbol{r} \\ \boldsymbol{A}_z\cdot\boldsymbol{r} \end{pmatrix} = \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} \tag{1-9}$$

Often we wish to solve for **r** in (1-7). While it seems that a simple matrix division would be sufficient, it turns out that matrix division cannot be easily defined. Instead, we introduce the concepts of the “identity matrix,” $\boldsymbol{I}$, and the “inverse matrix,” $\boldsymbol{A}^{-1}$. By definition

$$\boldsymbol{I}\,\boldsymbol{A} = \boldsymbol{A}\,\boldsymbol{I} = \boldsymbol{A} \qquad \boldsymbol{I} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \tag{1-10}$$

$$\boldsymbol{A}^{-1}\boldsymbol{A} = \boldsymbol{A}\,\boldsymbol{A}^{-1} = \boldsymbol{I}$$

Now we can schematically solve for $\boldsymbol{r}$:

$$\begin{gathered} \boldsymbol{A}\,\boldsymbol{r} = \boldsymbol{b} \\ \boldsymbol{A}^{-1}\boldsymbol{A}\,\boldsymbol{r} = \boldsymbol{A}^{-1}\boldsymbol{b} \\ \boldsymbol{r} = \boldsymbol{A}^{-1}\boldsymbol{b} \end{gathered} \tag{1–11}$$

We said “schematically” because, as of yet, we have no idea what $\boldsymbol{A}^{-1}$ is. In fact, we will leave the process of finding an inverse in general to a course in linear algebra. However, we can find the inverse of a few important matrices from physical principles. The second difficulty in actually applying (1-10) and (1-11) is that we have not yet defined what we mean by multiplying matrices together. This we shall now proceed to do.

1–2. Matrix Multiplication

Let $\boldsymbol{A}\,\boldsymbol{r} = \boldsymbol{b}$, where $\boldsymbol{b}$ is a column vector. We can also let $\boldsymbol{C}\,\boldsymbol{b} = \boldsymbol{d}$ where $\boldsymbol{C}$ is a matrix and $\boldsymbol{d}$ is a column vector. We see from the first equation that $\boldsymbol{d}$ has linear dependence on $\boldsymbol{r}$, so we may write it, in turn, as the product of some matrix, $\boldsymbol{E}$, and $\boldsymbol{r}$. That is:

$$\begin{gathered} \boldsymbol{C}\,\boldsymbol{b} = \boldsymbol{d} \\ \boldsymbol{C}\,\boldsymbol{A}\,\boldsymbol{r} = \boldsymbol{E}\,\boldsymbol{r} \\ \boldsymbol{C}\,\boldsymbol{A} = \boldsymbol{E} \end{gathered}$$

(1-12)

The algebra is a bit tedious, but it may be useful to write it out in detail.

$$\begin{pmatrix} C_{xx} & C_{xy} & C_{xz} \\ C_{yx} & C_{yy} & C_{yz} \\ C_{zx} & C_{zy} & C_{zz} \end{pmatrix} \begin{pmatrix} b_x \\ b_y \\ b_z \end{pmatrix} = \begin{pmatrix} C_{xx}b_x + C_{xy}b_y + C_{xz}b_z \\ C_{yx}b_x + C_{yy}b_y + C_{yz}b_z \\ C_{zx}b_x + C_{zy}b_y + C_{zz}b_z \end{pmatrix}$$

$$= \begin{pmatrix} C_{xx}(A_{xx}x + A_{xy}y + A_{xz}z) + C_{xy}(A_{yx}x + A_{yy}y + A_{yz}z) + C_{xz}(A_{zx}x + A_{zy}y + A_{zz}z) \\ C_{yx}(A_{xx}x + A_{xy}y + A_{xz}z) + C_{yy}(A_{yx}x + A_{yy}y + A_{yz}z) + C_{yz}(A_{zx}x + A_{zy}y + A_{zz}z) \\ C_{zx}(A_{xx}x + A_{xy}y + A_{xz}z) + C_{zy}(A_{yx}x + A_{yy}y + A_{yz}z) + C_{zz}(A_{zx}x + A_{zy}y + A_{zz}z) \end{pmatrix}$$

$$= \begin{pmatrix} (C_{xx}A_{xx} + C_{xy}A_{yx} + C_{xz}A_{zx})x + (C_{xx}A_{xy} + C_{xy}A_{yy} + C_{xz}A_{zy})y + (C_{xx}A_{xz} + C_{xy}A_{yz} + C_{xz}A_{zz})z \\ (C_{yx}A_{xx} + C_{yy}A_{yx} + C_{yz}A_{zx})x + (C_{yx}A_{xy} + C_{yy}A_{yy} + C_{yz}A_{zy})y + (C_{yx}A_{xz} + C_{yy}A_{yz} + C_{yz}A_{zz})z \\ (C_{zx}A_{xx} + C_{zy}A_{yx} + C_{zz}A_{zx})x + (C_{zx}A_{xy} + C_{zy}A_{yy} + C_{zz}A_{zy})y + (C_{zx}A_{xz} + C_{zy}A_{yz} + C_{zz}A_{zz})z \end{pmatrix}$$

$$= \begin{pmatrix} C_{xx}A_{xx} + C_{xy}A_{yx} + C_{xz}A_{zx} & C_{xx}A_{xy} + C_{xy}A_{yy} + C_{xz}A_{zy} & C_{xx}A_{xz} + C_{xy}A_{yz} + C_{xz}A_{zz} \\ C_{yx}A_{xx} + C_{yy}A_{yx} + C_{yz}A_{zx} & C_{yx}A_{xy} + C_{yy}A_{yy} + C_{yz}A_{zy} & C_{yx}A_{xz} + C_{yy}A_{yz} + C_{yz}A_{zz} \\ C_{zx}A_{xx} + C_{zy}A_{yx} + C_{zz}A_{zx} & C_{zx}A_{xy} + C_{zy}A_{yy} + C_{zz}A_{zy}) & C_{zx}A_{xz} + C_{zy}A_{yz} + C_{zz}A_{zz} \end{pmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix}$$

This then implies that

$$\boldsymbol{E} = \boldsymbol{C}\,\boldsymbol{A} = \begin{pmatrix} C_{xx}A_{xx} + C_{xy}A_{yx} + C_{xz}A_{zx} & C_{xx}A_{xy} + C_{xy}A_{yy} + C_{xz}A_{zy} & C_{xx}A_{xz} + C_{xy}A_{yz} + C_{xz}A_{zz} \\ C_{yx}A_{xx} + C_{yy}A_{yx} + C_{yz}A_{zx} & C_{yx}A_{xy} + C_{yy}A_{yy} + C_{yz}A_{zy} & C_{yx}A_{xz} + C_{yy}A_{yz} + C_{yz}A_{zz} \\ C_{zx}A_{xx} + C_{zy}A_{yx} + C_{zz}A_{zx} & C_{zx}A_{xy} + C_{zy}A_{yy} + C_{zz}A_{zy}) & C_{zx}A_{xz} + C_{zy}A_{yz} + C_{zz}A_{zz} \end{pmatrix}$$

$$= \begin{pmatrix} \boldsymbol{C^T}_x \boldsymbol{a_x} & \boldsymbol{C^T}_x \boldsymbol{a_y} & \boldsymbol{C^T}_x \boldsymbol{a_z} \\ \boldsymbol{C^T}_y \boldsymbol{a_x} & \boldsymbol{C^T}_y \boldsymbol{a_y} & \boldsymbol{C^T}_y \boldsymbol{a_z} \\ \boldsymbol{C^T}_z \boldsymbol{a_x} & \boldsymbol{C^T}_z \boldsymbol{a_y} & \boldsymbol{C^T}_z \boldsymbol{a_z} \end{pmatrix} = \begin{pmatrix} \boldsymbol{C}_x \cdot \boldsymbol{a_x} & \boldsymbol{C}_x \cdot \boldsymbol{a_y} & \boldsymbol{C}_x \cdot \boldsymbol{a_z} \\ \boldsymbol{C}_y \cdot \boldsymbol{a_x} & \boldsymbol{C}_y \cdot \boldsymbol{a_y} & \boldsymbol{C}_y \cdot \boldsymbol{a_z} \\ \boldsymbol{C}_z \cdot \boldsymbol{a_x} & \boldsymbol{C}_z \cdot \boldsymbol{a_y} & \boldsymbol{C}_z \cdot \boldsymbol{a_z} \end{pmatrix} \qquad (1–13)$$

We see that every element of the product matrix is a dot product of one row of the first matrix in the product with one column of the second matrix in the product. The process can be tedious, but fortunately, most of the matrices we'll be using have zero for many of the elements. Otherwise, it is often useful to do matrix products (and other manipulations) with computers so as to minimize the chances of computational errors.

1–3. Matrices as Operators

Matrices are often useful to transform a vector or another matrix according to some prescription. We shall look at three specific types of transformations shortly; however, let us first make a few important generalizations about matrices.

The "diagonal" elements of a matrix are defined as the upper-left to lower-right components, such as A_{xx},

A_{yy}, and A_{zz}. A "diagonal matrix" is one whose off-diagonal components are all zero. Diagonal matrices leave x-components of a vector as x-components; they cannot take an x-component and turn it into a y-component. If the magnitude of a diagonal component of a matrix is 1, the corresponding component of a vector remains unchanged. If it is a positive number, it changes the size of the component. If it is a negative number, it also reverses the sign of the component. Note how a few diagonal matrices affect the vector $\boldsymbol{r}$:

$$\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}x\\y\\z\end{pmatrix}\qquad\begin{pmatrix}1&0&0\\0&2&0\\0&0&3\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}x\\2y\\3z\end{pmatrix}\qquad\begin{pmatrix}1&0&0\\0&1&0\\0&0&-1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}x\\y\\-z\end{pmatrix}$$

The off-diagonal elements, on the other hand, control the coupling between components. If the matrix element $A_{xy}=2$, for example, it multiplies the y-component of the initial vector by 2, and moves it to the x-component of the final vector. Note, then, how the following matrices work:

$$\begin{pmatrix}1&0&0\\0&0&1\\0&1&0\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}x\\z\\y\end{pmatrix}\qquad\begin{pmatrix}0&0&1\\2&0&0\\0&3&0\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}z\\2x\\3y\end{pmatrix}\qquad\begin{pmatrix}0&1&-1\\-1&0&1\\1&-1&0\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}y-z\\z-x\\x-y\end{pmatrix}$$

Let us now see how we can represent a few mathematical operations with matrices.

A. Mirror reflection

If an object is placed a distance x in front of a mirror, the image will be at a distance $-x$ from the mirror. The y and z coordinates of the image are the same as for the object. The matrix which performs mirror reflection is clearly:

$$\boldsymbol{M}_x\boldsymbol{r}=\begin{pmatrix}-1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}-x\\y\\z\end{pmatrix}$$

Physically, we know that the inverse of a mirror reflection should be the mirror reflection itself. Let's see if this works:

$$\boldsymbol{M}_x\boldsymbol{M}_x=\begin{pmatrix}-1&0&0\\0&1&0\\0&0&1\end{pmatrix}\begin{pmatrix}-1&0&0\\0&1&0\\0&0&1\end{pmatrix}=\begin{pmatrix}1&0&0\\0&1&0\\0&0&1\end{pmatrix}=\boldsymbol{I}$$

B. Rotation about the x-axis

Let's rotate a vector about the x-axis, so that what was initially the z-component becomes the y-component (and conversely, what was initially the y-component becomes the $-z$-component). The matrix which performs this transformation is:

$$\boldsymbol{R}_{+x}\boldsymbol{r}=\begin{pmatrix}1&0&0\\0&0&1\\0&-1&0\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix}=\begin{pmatrix}x\\z\\-y\end{pmatrix}\qquad(1\text{-}14)$$

The inverse of this should be a "backward rotation."

$$\boldsymbol{R}_{+x}\boldsymbol{R}_{-x} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & +1 \\ 0 & -1 & 0 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 \\ 0 & 0 & -1 \\ 0 & +1 & 0 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \boldsymbol{I}$$

C. Boost in the *z*-direction

Our last example will be to determine the relationship between the components of a vector measured in a coordinate system fixed to the ground and the components measured in a coordinate system fixed on a moving train. Let (x,y,z) be the vector measured in the ground system and let the train move with a velocity of v in the $+z$-direction. If at $t = 0$, the origins of the two coordinate systems coincide and the directions of the two sets of axes are the same, we see that the coordinates measured by the train must be $(x,y,z-vt)$. Notice that time now becomes an additional variable. We can then construct four-element vectors to include time. The matrix which accomplishes this transformation is just:

$$\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -v & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} t \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} t \\ x \\ y \\ z-vt \end{pmatrix}$$

Although there is nothing technically wrong with this, physicists like to have each element of a vector or matrix to have the same units. To accomplish this, we can multiply t by a constant, c, with dimensions of velocity. For simplicity, we can let c = 1 m/s. We can then rewrite the above equation as:

$$\boldsymbol{B}_z\boldsymbol{r} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -v/c & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} ct \\ x \\ y \\ z \end{pmatrix} = \begin{pmatrix} ct \\ x \\ y \\ z-vt \end{pmatrix} \tag{1-15}$$

Note that the lower-left component is non-zero because the *z* position of the point as measured on the train depends on the time. The upper-right component is 0, however, because time does not depend on position.

The inverse of this matrix must then be the same matrix with v replaced by $-v$.

$$\boldsymbol{B}_{+z}\boldsymbol{B}_{-z} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ -v/c & 0 & 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ +v/c & 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} = \boldsymbol{I}$$

1–4. Combining Operations

Let us now see what happens when we combine operations. Let us first do a mirror reflection in the *z* direction, then do a rotation about the *x*-axis:

$$\boldsymbol{R}_{+x}(\boldsymbol{M}_{+z}\boldsymbol{r}) = \boldsymbol{R}_{+x}\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & -1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & -1 & 0\end{pmatrix}\begin{pmatrix}x\\y\\-z\end{pmatrix} = \begin{pmatrix}x\\-z\\-y\end{pmatrix}$$

Matrix multiplication is associative so that we could have multiplied the matrices together first to create a single matrix to act on ***r***.

$$(\boldsymbol{R}_{+x}\boldsymbol{M}_{+z})\,\boldsymbol{r} = \begin{pmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & -1 & 0\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & -1\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\0 & 0 & -1\\0 & -1 & 0\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}x\\-z\\-y\end{pmatrix}$$

On the other hand, matrix multiplication is **not** commutative. If we first do the rotation, then the reflection, we have:

$$(\boldsymbol{M}_{+z}\boldsymbol{R}_{+x})\,\boldsymbol{r} = \begin{pmatrix}1 & 0 & 0\\0 & 1 & 0\\0 & 0 & -1\end{pmatrix}\begin{pmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & -1 & 0\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}1 & 0 & 0\\0 & 0 & 1\\0 & 1 & 0\end{pmatrix}\begin{pmatrix}x\\y\\z\end{pmatrix} = \begin{pmatrix}x\\z\\y\end{pmatrix}$$

1–6. Matrix Transformations (optional)

Another thing that is often useful is to transform matrices to new coordinate systems. To see how this is done, we can take a specific example. We know the matrix for a boost in the *z*-direction, (1-15), and we know how to construct a rotation matrix, ***R***, that changes the *z*-axis into the *y*-axis (1-14). Let us then combine these results to obtain the matrix for a boost in the *y* -direction. (We do need to make a slight modification to the rotation matrix to accommodate the four dimensional vector, but this is simple because rotation is unrelated to the time coordinate.) That is, we take a vector, boost it in the *z* direction, then rotate it so that the *z* direction turns into the *y* direction. We let a tilde represent a quantity in the rotated coordinate system and the subscript *B* indicate the "boosted" vector. Symbolically, this is:

$$\begin{aligned}\tilde{\boldsymbol{r}}_{\boldsymbol{B}} &= \boldsymbol{R}_{+x}\boldsymbol{B}_{+z}\boldsymbol{r}\\ &= \boldsymbol{R}_{+x}\boldsymbol{B}_{+z}\boldsymbol{I}\,\boldsymbol{r}\\ &= \boldsymbol{R}_{+x}\boldsymbol{B}_{+z}(\boldsymbol{R}_{-x}\boldsymbol{R}_{+x})\,\boldsymbol{r}\\ &= \boldsymbol{R}_{+x}\boldsymbol{B}_{+z}\boldsymbol{R}_{-x}\tilde{\boldsymbol{r}}\end{aligned}$$

This is equivalent to first rotating ***r*** and then boosting it in the *y* direction.

$$\tilde{\boldsymbol{r}}_{\boldsymbol{B}} = \tilde{\boldsymbol{B}}_{+y}\tilde{\boldsymbol{r}}$$

Since these vectors are equal, we have:

$$\tilde{\boldsymbol{B}}_{+y} = \boldsymbol{R}_{+x}\boldsymbol{B}_{+z}\boldsymbol{R}_{-x}.$$

We can explicitly evaluate this matrix:

$$\tilde{\boldsymbol{B}}_{+y} = \begin{pmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & 1\\0 & 0 & -1 & 0\end{pmatrix}\begin{pmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 1 & 0\\-v/c & 0 & 0 & 1\end{pmatrix}\begin{pmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & -1\\0 & 0 & 1 & 0\end{pmatrix} = \begin{pmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\-v/c & 0 & 0 & 1\\0 & 0 & -1 & 0\end{pmatrix}\begin{pmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\0 & 0 & 0 & -1\\0 & 0 & 1 & 0\end{pmatrix} = \begin{pmatrix}1 & 0 & 0 & 0\\0 & 1 & 0 & 0\\-v/c & 0 & 1 & 0\\0 & 0 & 0 & 1\end{pmatrix}$$

This is clearly the form we should expect for a boost in the *y*-direction.

In this particular example, we could have guessed the answer; however, this procedure is often very useful for generating new transformation matrices from known matrices. In general, we may write this result as:

$$\tilde{\boldsymbol{A}} = \boldsymbol{T}\,\boldsymbol{A}\,\boldsymbol{T}^{-1}. \tag{1-16}$$

This is a very brief introduction to linear algebra, but it does give us some tools to make the study of relativity somewhat simpler.