A Kinematic Approach to Special Relativity

Notation

Since we will be dealing with three and four-dimensional vectors, matrices, magnitudes, components, etc., the notation can sometimes become confusing. We will use the following rules:

Special quantities:

c	Non-relativistically, an arbitrarily chosen constant with units of velocity. Relativistically, the speed of light.
$\boldsymbol{\beta}$	$\boldsymbol{v} / c$. Note that this is a dimensionless vector.
γ	$1/\sqrt{1-\beta^2}$

Special vectors:

$\boldsymbol{r}$	The position vector in three dimensions.
$\boldsymbol{x}$	The space-time vector or four-position.
$\boldsymbol{p}$	The momentum vector in three dimensions.
$\boldsymbol{P}$	The propagator.
$\boldsymbol{E}$	The energy-momentum vector or four-momentum.
$\boldsymbol{F}$	The force vector in three dimensions.
$\boldsymbol{f}$	The force vector in four dimensions or four-force.

General notation:

$\boldsymbol{a}^3$	A three-dimensional vector
$\boldsymbol{a}^4$	A four-dimensional vector
$\boldsymbol{a}^{\mathrm{T}}$	The transpose of a vector or matrix
$\boldsymbol{A}$	A matrix (upper case, bold)
a_x	The *x* component the vector **a**.
A_{xx}	The *xx* element of the matrix **A**.

Introduction – The Beginnings of Special Relativity

In 1905, Albert Einstein wrote his landmark paper, "On the Electrodynamics of Moving Bodies," in which the Special Theory of Relativity was first postulated. Einstein based his derivation of relativity on two fundamental postulates:

1) Galilean Relativity: The laws of physics must be the same for all non-accelerating observers. This means that the mathematical form of all physical laws is independent of the motion of an observer as long as the observer does not accelerate. This principle had been accepted from Newton's time. (Although Galileo roughly formulated the concept of Galilean relativity, it was first clearly stated by Newton.)

2) Constant speed of light: The speed of light in vacuum is a constant, independent of the motion of either the source or the observer. This was an experimental observation made in 1887 by Albert Michelson and Robert Morley in their famous interferometry experiment. This means, for example, that if a spaceship flies past a lighthouse at almost the speed of light, the passengers would measure the velocity of the light emitted by the lighthouse to be exactly the same as an observer in the lighthouse.

We must remember that, although there was good reason to accept these postulates, they are still postulates. That is, they are premises which cannot be proven.

Because the constant velocity of light was completely at odds with Newtonian physics and seemed illogical, many people were slow to accept relativity. This was compounded by the fact that if relativity were true, many fundamental concepts of space, time, and energy would have to be revised.

Even before Einstein proposed special relativity, however, others were noticing problems with the traditional laws of physics. One of these was the Dutch physicist H. A. Lorentz. Lorentz, one the foremost experts on electromagnetic theory, noticed several inconsistencies between predictions based on Maxwell's Equations and on Newtonian dynamics. In an effort to reconcile these, he proposed in 1904 what we now call the "Lorentz transformations" of position and time coordinates. These transformations form the heart of special relativity. In fact, the very existence of the magnetic field is a consequence of special relativity embedded in Maxwell's Equations.

Time has demonstrated conclusively that Einstein was right. In modern particle accelerators, fundamental particles are brought to velocities nearly the speed of light. Their behavior is precisely predicted by special relativity.

Aside from electromagnetic theory, the practical applications of special relativity are mostly confined to subatomic physics. For this reason, we will depart from the traditional derivation of relativity in favor of an approach based on kinematics and symmetry. We will later come back and see that the same results can be obtained by starting with the Two Postulates of Relativity.

Before we begin our discussion of relativity, however, we will start with a chapter on the basics of linear algebra.