Lesson 11 – Faraday's Law of Induction

11.0 Introduction

In the last lesson, we introduced Faraday's law of induction in several forms. In words, we have:

Faraday's Law of Induction

The electric line integral around an Amperian loop is proportional to the change in the number of magnetic field lines passing through the loop.

In addition, we wrote Faraday's law in both differential and integral forms:

Faraday's Law, Differential Form

$$\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

Faraday's Law, Integral Form

$$\Lambda_E = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt}\Phi_{B,enc}$$

Faraday's law is one of the most important ideas we're going to learn in this course, so in this lesson we'll consider just what this law means and how to apply it. But first, we want to introduce the concept of electromotive force.

11.1 Electromotive Force

"Electromotive force" is an old term for the voltage that pushes current around a circuit. It is *not* a force at all, but a voltage. We would probably do well to ignore the term completely, except for the fact that it is still widely used today. Usually, we just call it "EMF" and represent it with a script E. Because of confusion in fonts, we will use the Greek letter epsilon, ε, and hope that confusion with ε_0 does not ensue.

There are two basic types of EMF. The first, such as a battery, produces a constant voltage that does not change in time. We simply called this EMF the "voltage" in previous lessons. Other sources, include Mastering Physics, sometimes call the voltage of a battery and "EMF" to distinguish it from the voltage drop across a resistor, for example. A second kind of

EMF is an "induced EMF." An induced EMF is the voltage produced by changing magnetic fields (or changing magnetic flux) – that is, it's Faraday's law voltage. You're already familiar with the EMF of batteries and we'll still call this "voltage" most of the time. But it is useful to discuss a few peculiarities of induced voltage before we move on.

Consider a loop of wire of radius R. We will assume that this wire has an electric field of magnitude E going around it, as shown in Fig 11.1.
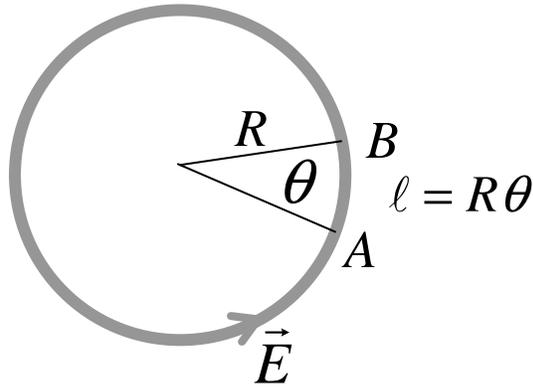

Fig. 11.1 A loop of wire with an induced electric field.

From the definition of voltage, we can find an expression for the change of voltage between points A and B in Fig. 11.1.

$$\Delta V = -\int_A^B E d\ell = -ER\theta$$

If we go all the way around the loop from A back to A again, then we have $\Delta V - 2\pi RE$. However, we would expect the change in voltage around a closed path to be zero. The problem is that potential energy can't be defined for looping electric fields.

On the other hand, we can think of the wire segment from point A to point B as a battery that helps drive current counterclockwise around the loop, as shown in Fig. 11.2.
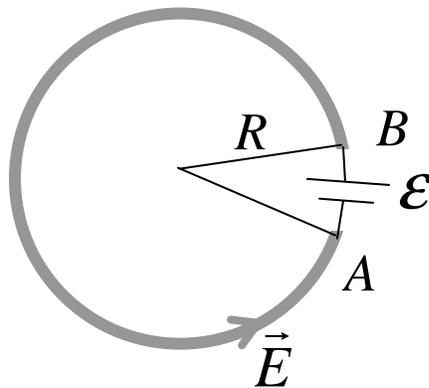

Fig. 11.2 A wire segment with induced EMF is equivalent to a battery.

In this case, the work done by the electric field is the EMF,

$$\varepsilon = +\int_A^B Ed\ell = +ER\theta$$

So the only difference between EMF and ΔV is the sign. More generally, EMF is defined by:

$$\varepsilon = +\int_A^B \vec{E} \cdot d\vec{\ell} = \Lambda_E (A \to B).$$

Things to remember:
• When electric field form loops, the usual ideas of potential energy and energy conservation cannot be applied.
• The line integral of the electric field is called the EMF. EMF is the voltage that is caused by looping electric fields. It behaves in much the same way as the voltage of a battery.

11.2 EMF without Acceleration.

As we learned in Lesson 10, induced currents can be produced by accelerating charges. One example of this is shown in Fig. 11.3. Here an induced current in a loop is produced by increasing the current going a long, straight wire located near the loop.
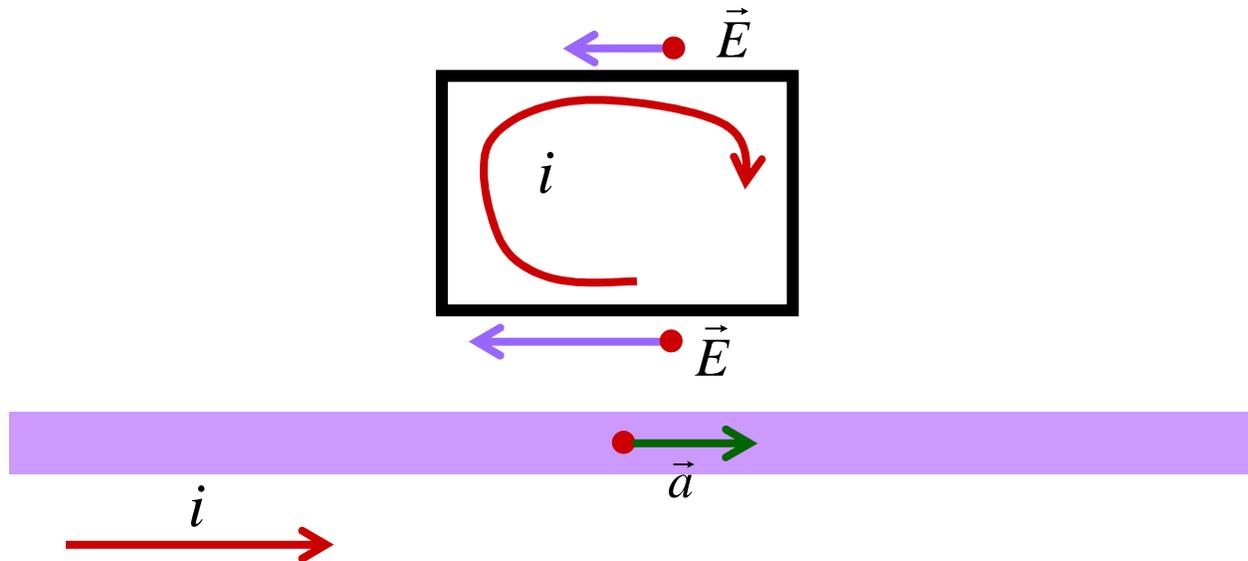
Figure 11.3 When current in the wire increases, an induced current will flow through the loop.

However, not all induced currents are due to acceleration. While we are in the process of moving a conducting loop nearer to a long wire carrying a constant current i the Lorentz force produces a force on charges in the wire: $\vec{F} = q\vec{v} \times \vec{B}$. Since the magnetic field is coming out of

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the screen, the direction of the force on each positive current carrier is toward the left. However, the magnetic field is stronger near the wire, so the force on the bottom segment of the loop wins out and current travels clockwise through the loop. This is illustrated in Fig. 11.4.
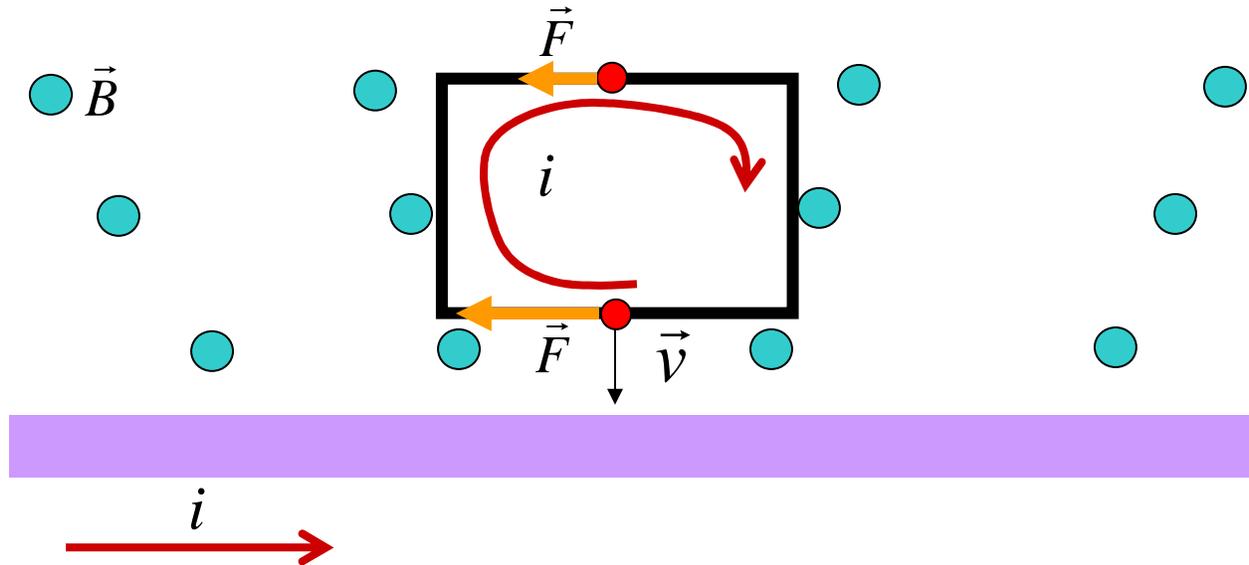

Figure 11.4. Induced current in a wire moving toward a wire carrying constant current.

If an observer at rest with respect to the loop views this same process, she sees a wire moving toward her at constant speed. However, she also sees a current passing through the wire. If the charges in the wire are stationary, the magnetic field of the wire can't exert a force on the charges to start them moving. We conclude that the uniformly-moving wire must be producing an electric field that drives charge around the loop.

If we go back to threads and stubs, we can see the reason for this. Remember that a fine balance between the electric field of stationary charges and the moving valence electrons gives zero electric field for stationary wires. But when the velocity of a wire is perpendicular to the current, this balance is disrupted.

Things to remember:
• Induced current can be produced my "motional EMF" as well as by acceleration fields.
• Motional EMF is the force on charge carriers in a current caused by a wire moving through a magnetic field.

11.3 Faraday's Law

So looping electric fields can be produced by accelerating charges or they can be produced by charges in constant motion. As might be imagined, it is often very difficult to calculate these kinds of fields for wires, loops, and magnets in arbitrary relative motion. But then, it was also very difficult to find the electric for a distribution as simple as a stationary cube with uniform charge density. However, by using Ampere's law and Gauss's law, we were able to find fields for a few symmetric distributions of charges and currents. As it turns out, we can find a number of important results by using Faraday's law.

We first might ask if there's anything in common between the cases of motional EMF and the EMF of accelerating charges. As you can verify by a little thought, one thing that changes in both cases is number of magnetic field lines going through the Amperian loop. In Lesson 10, we suggested that for accelerating charges, there was a relatively simple relationship between the electric and magnetic flux given by Faraday's law:

Faraday's Law in Integral Form

(11.1)
$$\Lambda_E = \varepsilon = \oint \vec{E} \cdot d\vec{\ell} = -\frac{d}{dt}\Phi_{B,enc} = -\frac{d}{dt}\int B \cdot d\vec{A}$$

where

Λ_E is the line integral of the electric field around a closed Amperian loop in volts (V).

ε is the EMF around the Amperian loop. It is equivalent to the line integral.

The line integral is an taken around a complete Amperian loop.

$\Phi_{B,enc}$ is the net magnetic flux passing through the Amperian loop.

The surface integral is taken over any surface bounded by the Amperian loop.

We will usually use Faraday's law in the simple form:

$$(11.2) \qquad \varepsilon = -\frac{d}{dt}\Phi_{B,enc}$$

Typically, a Faraday's law problem is to find the EMF around a loop by determining how magnetic flux through the loop changes in time.

Although we won't use it much, we also have the differential form of Faraday's law that tells us how the spatial variation of the electric field is related to the time variation of the magnetic field at a point in space.

Faraday's Law in Differential Form

$$(11.2) \qquad \nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$$

which means:
At a point in space, the curl of the electric field is minus the time rate of change of the magnetic field at the same point.

As it turns out, Faraday's law applies equally well to looping electric fields caused by relative motion or by accelerating particles.

Things to remember:
• Faraday's law tells us that the voltage around a loop is minus the rate at which magnetic flux through the loop changes. $\varepsilon = -\frac{d}{dt}\Phi_{B,enc}$.
• Faraday's law tells us that the curl of the electric field at a point in space is minus the rate at which the magnetic field at the point changes. $\nabla \times \vec{E} = -\frac{\partial \vec{B}}{\partial t}$

11.4 Induction Experimentally

The best way to understand Faraday's law is look at some specific example. Probably the easiest way to induce a current is bring a magnet near a loop of wire. The one complication to this is that the magnetic field of permanent magnets is a little complicated. The field lines of a typical bar magnet are shown in Fig. 11.5.
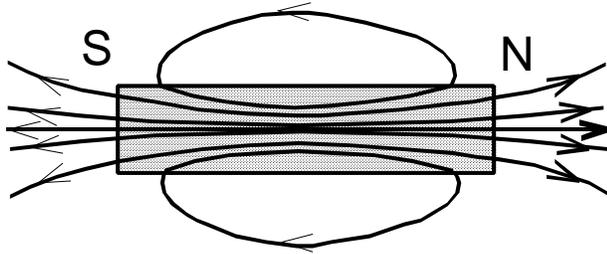
Figure 11.5. The field lines of a bar magnet.

The field of permanent magnets is produced by electrons orbiting in atoms. Magnetic field lines are concentrated within the permanent magnet itself. Permanent magnets can be made in various shapes, but the field lines will always travel from the south pole to the north pole within the magnet. Outside the magnet, the field lines will go from north pole to south pole, forming closed loops. Note that the field is stronger near the poles of the magnet.

In the early 19th Century many characteristics of electricity and magnetism were being discovered in a flurry of experimentation. It was known that wires produce magnetic fields, so it was natural to ask what connections there were between electricity and magnetism. In 1831 Michael Faraday discovered that magnetic fields can produce electric current is wires. The easiest sort of experiment to visualize is depicted in Fig. 11.6. As a permanent magnet is moved into or out of a circular loop of wire, a current flows in the loop and a light bulb lights up. We can use meters to measure the voltage across the light bulb and to measure the current in the loop.
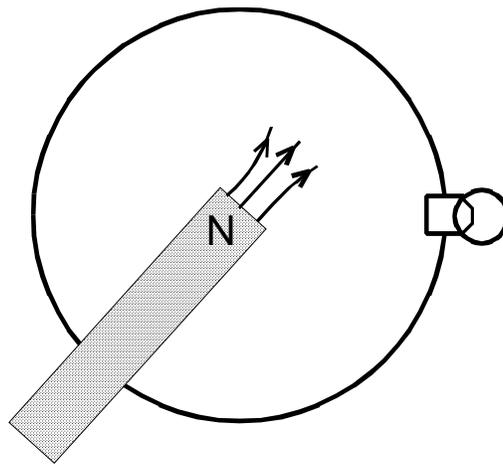

Figure 11.6. A simple induction experiment.

We can summarize the results of a number of experiments as follows:
• When the magnet is stationary, there is no voltage.
• The voltage is twice as large when the magnet moves twice as fast.
• The voltage is twice as large when the magnetic field is twice as strong.
• If a small loop is moved into a large region of magnetic field, the voltage is twice as large when the loop has twice the area.
• The voltage is less when the loop is tilted so that it is not perpendicular to the magnet.

Think About It

Explain why each of these results is consistent with Faraday's law, $\varepsilon = -\dfrac{d}{dt}\Phi_{B,enc}$.

One thing we did not mention in the above paragraphs was the direction in which current flows in the loop. The rule to determine this is called Lenz's law. Simply put, Lenz's law is: "Nature does not like change." When the magnetic flux through a conducting loop changes, current is produced in the loop. The induced current in turn produces an induced magnetic field. The induced current flows in such a direction that the induced magnetic field opposes change in the flux. To make the meaning of Lenz's law clear, let us illustrate it with a few examples.

Lenz's Law

The induced current in a wire goes in the direction that opposes change in the external magnetic field. If there is no wire present, the induced electric field is in the direction that current would flow if there were a wire.

First, consider the loop shown in Fig. 11.6. Let us move a bar magnet into the loop with its north pole facing the screen. When the bar magnet is far away, the magnetic flux through the loop is zero. Nature wants to keep the magnetic flux through the loop zero. As the magnet approaches the loop, magnetic field lines pointing into the screen enter the loop. The loop then produces an induced current which in turn produces an induced magnetic field. The induced magnetic field points out of the screen so as to oppose the **change** in the external field. By using the right-hand rule for the magnetic field of a loop, we see that the current must flow in a counterclockwise direction.

Next, after the magnet is in the loop, let us pull it back out. In this case there is a magnetic flux through the loop, so nature wants to maintain that flux. As the magnet is pulled out, the magnetic field lines from the north pole of the magnet are still into the screen; however, the strength of the magnetic field is decreasing. The induced magnetic field must then be into the

screen, so as to oppose **change** in the external field. Note that in this case, the induced field is in the same direction as the external field. To produce an induced magnetic field into the screen, the current must be clockwise.

The minus sign in Faraday's law expresses this tendency to oppose change. We could give a series of rules for positive and negative directions of current flow, etc., so that the minus sign would properly give the direction of current flow; however, for most applications it is easier to forget the minus sign and determine the direction of the induced current by applying Lenz's law qualitatively.

Things to remember:
• We can create an induced electric field by changing the flux through a loop in a number of ways: change the size of the loop, change the strength of the magnetic field, or change the orientation of the loop with respect to field.
• Lenz's law states that the direction of the induced field can be determined from the direction of the induced current that would oppose *change* in the external field.
• Mathematically, Lenz's law is represented by the "–" sign in Faraday's law.

11.5 Using Lenz's Law

It is useful to try a few examples to see how Lenz's law can easily give us the direction of induced electric fields.

Example 11.1. A loop around the screen.

Imagine a loop of wire going out around your screen. Take a bar magnet with its north pole pointing toward the screen. Move the bar magnet toward the screen. Which way does induced current flow around the loop?

We always need to consider two things when we apply Lenz's law: 1) What is the direction of the external (not induced) magnetic field? 2) Is the external magnetic field increasing or is it decreasing. Once we have answered these questions, all we need to do is decide which direction induced current must flow in order to oppose *change* in the field through the loop.

Here, the magnetic field from the bar magnet points into the screen and is increasing. To oppose the increase of the magnetic field into the screen, the induced magnetic field must come out of the screen. By the right-hand rule, current must flow counterclockwise around the loop to produce a magnetic field out of the screen.

What happens if we move the same magnet away from the screen? The magnetic field from the bar magnet still points into the screen, but it is decreasing. To oppose the decrease of the magnetic field into the screen, the induced magnetic field must also go into the screen. By the right-hand rule, current must flow clockwise around the loop to produce this induced magnetic field.

Example 1.2. Loop around an electromagnet.

An electromagnet has a loop of wire around it as shown in Fig. 11.7.
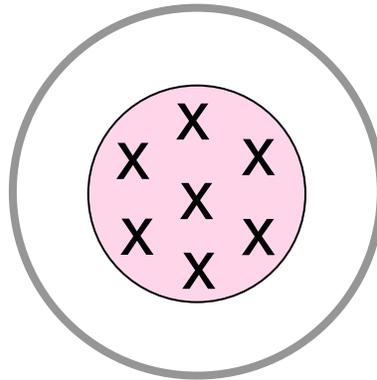
Fig, 11.7. A loop of wire around an electromagnet.

If the current passing into the electromagnet is decreasing, in which way does current flow through the circular loop?

The magnetic field points into the screen. If current is decreasing, the magnetic field, in turn, is decreasing. In order to keep this external field from decreasing, there must be an induced field also pointing into the screen. Hence, current flows clockwise through the loop.

If the wire is not conducting, what kind of field is induced around the loop? In what direction does the field point?

Even without a wire, an induced electric field is produced. To determine the direction of the field, just ask what direction current would flow if there a wire. The electric field lines, then, are loops going clockwise around the electromagnet.

Things to remember:
• Be sure you know how to find the direction of induced currents and electric fields by using Lenz's law.

11.6. Using Faraday's Law

Simply put, Faraday's law states that EMF around a loop equals the rate at which magnetic flux though the loop changes. Magnetic flux in turn is given by the expression:

$$\Phi_M = \int \vec{B} \cdot d\vec{A}.$$

We will only be interested in problems where the magnetic field is uniform over the loop so that the flux simplifies to

$$\Phi_B = \vec{B} \cdot \vec{A} = BA\cos\theta$$

where

B is the magnetic field strength in tesla (T).

A is the area of the loop in square meters (m^2).

θ is the angle between the magnetic field and the loop's normal.

Since the EMF equals the rate at which the magnetic flux changes, we can induce an EMF by changing the magnetic field, by changing the area of the loop, or by changing the orientation of the loop with respect to the magnetic field. We shall now look at each of three cases in turn.

Example 11.3. Changing Orientation

A circular loop of radius R is placed in a uniform magnetic field, B. The loop is connected to a handle so that the loop can be rotated about an axis perpendicular to the magnetic field. The loop is rotated with and angular velocity ω.
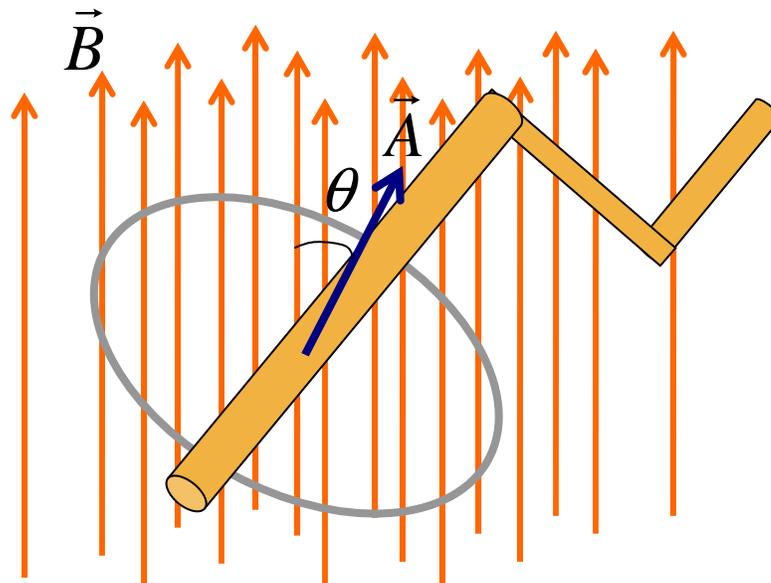

Figure 11.8. A loop rotating in uniform magnetic field.

To apply Faraday's law to this problem, all we need to do is identify the magnetic flux and take the derivative of that flux. The only trick to this problem is to identify θ in terms of ω, which is given. The needed relation is just $\theta = \omega t$. Then:

$$\varepsilon = -\frac{d}{dt} BA\cos\theta = -B\pi R^2 \frac{d}{dt}\cos\omega t = B\pi R^2 \omega\sin\omega t .$$

Note that the EMF varies sinusoidally in time and that its amplitude is proportional to the magnetic field, the loop area, and the angular velocity.

Example 11.4. Changing Area

A u-shaped wire is placed in a magnetic field as shown in Fig. 11.9. A fourth, moveable segment containing a resistor of resistance R is placed across the u. This segment is moved to left with a velocity v.
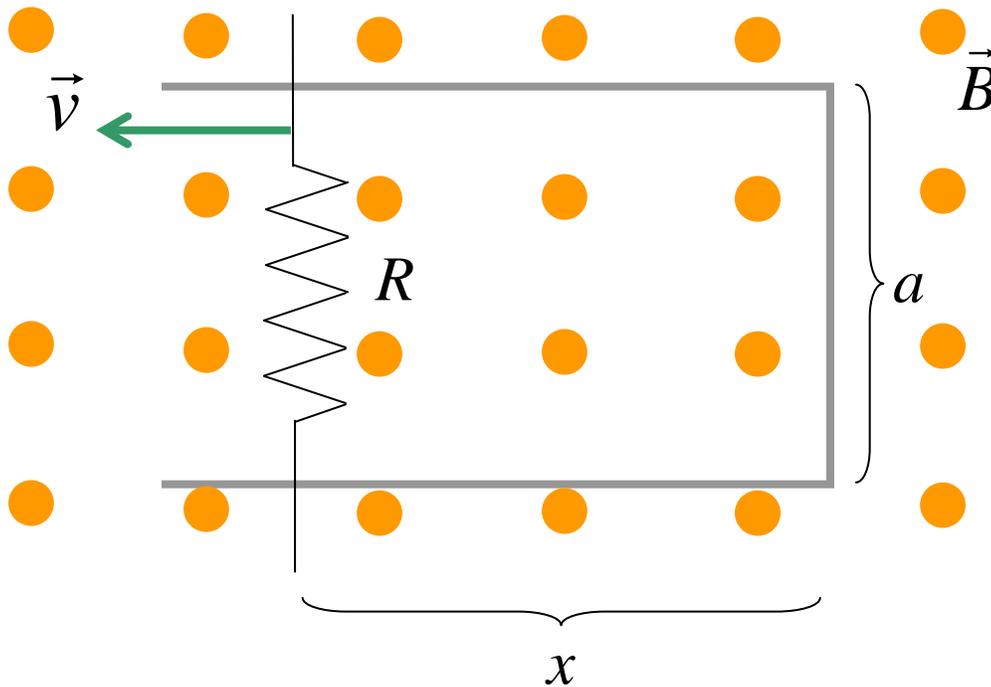
Figure 11.9. A loop of variable size.

(a) Find the EMF around the loop.

$$\varepsilon = -\frac{d\Phi_B}{dt} = -Ba\frac{dx}{dt} = -Bav$$

(b) Find the direction of the induced current.

The magnetic field is coming out of the screen. The flux is increasing. In order to oppose this change in flux, the induced magnetic field must be into the screen. hence, the induced current must go clockwise around the loop.

(c) Find the force on the moveable section of wire.

The force is given by $\vec{F} = i\vec{\ell} \times \vec{B}$. The current in this section of wire is directed upward, so the force is to right.

We also need to know what the current is. Because of the resistor in the wire, the entire EMF (or nearly so) is across the resistor, so $\varepsilon = iR$.

$$F = i\ell B = \frac{\varepsilon}{R} aB = \frac{v}{R} a^2 B^2$$

(d) Find the power dissipated in the resistor.

$$P = IV = i\varepsilon = \frac{\varepsilon^2}{R} = \frac{(vaB)^2}{R}$$

(e) Find the mechanical power provided to the moveable wire segment.

The mechanical power is $\dfrac{d}{dt} W = F \dfrac{dx}{dt} = Fv$.

$$P = \frac{v}{R} a^2 B^2 v = \frac{(vaB)^2}{R}$$

Example 11.5. Increasing B field.

      An electromagnet has a uniform magnetic field between two circular pole faces of radius R. A circular loop of wire of radius r is placed inside the magnetic field. A magnetic field going into screen varies in time as $B = \alpha t$ where α is a positive constant.
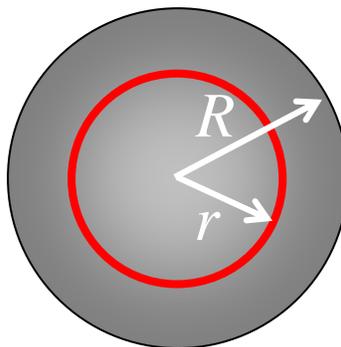

Figure 11.10. A loop in a circular magnetic field.

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(a) Find the EMF around the loop.

The flux inside the loop is $\Phi_B = \pi r^2 B$, so the EMF is:

$$\varepsilon = -\pi r^2 \frac{d}{dt} B = -\pi r^2 \alpha$$

The – sign only indicates direction, but that's most easily obtained from Lenz's law anyway.

(b) In what direction does current flow through the loop?

The external field is into the screen and increasing. The internal field must be out of the screen to oppose change. The current must be counterclockwise.

(c) What is the induced electric field in the wire?

This is one case where symmetry will actually allow us to find the electric field. We know the line integral reduces to $\Lambda_E = 2\pi r E$, much as it does with Ampère's law. Hence:

$$\varepsilon = -\pi r^2 \alpha = 2\pi r E$$

$$E = -\frac{\alpha r}{2}$$

The direction of the electric field is counterclockwise, as is the current.

(d) If we remove the wire, what is the electric field at radius r?

The electric field is the same whether or not the wire is present.

Things to remember:
• Be sure you know how to do Faraday's law problems when 1) the B field varies in time, 2) the loop area varies in time, or 3) the orientation of the loop varies in time.
• Be able to find the electric field inside and outside a circular magnetic field that is changing in time.

11.7 Eddy Currents

Whenever we move a conductor into a magnetic field or out of a magnetic field, currents are induced in the conductor. For example, let's push a circular disk into a region where there is a magnetic field, as shown in Fig. 11.11. Such currents are called "eddy currents."
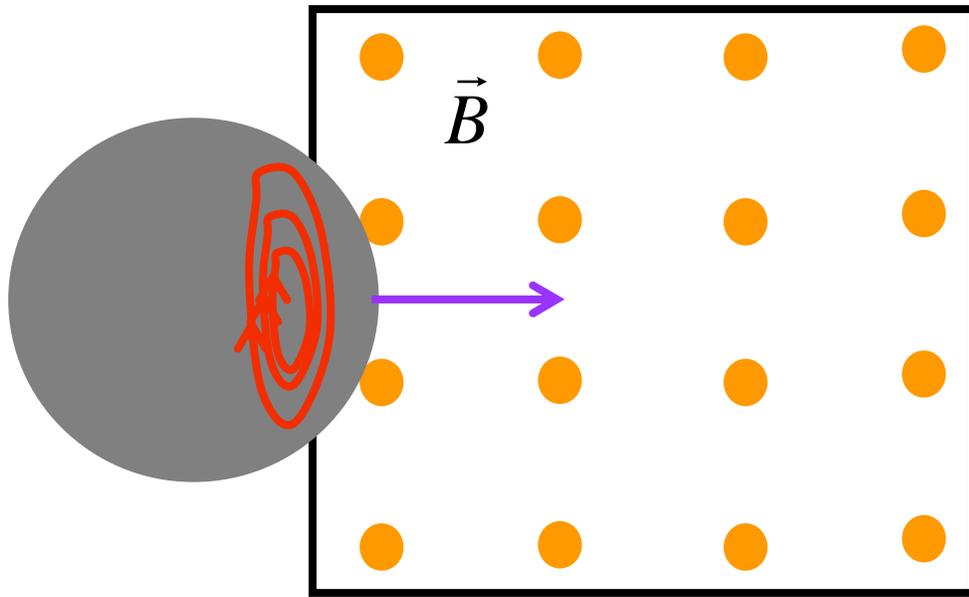
Figure 11.11. Eddy currents produced in a conducting disk.

If we look at this same process from the edge, we can see that the disk becomes an electromagnet. The north pole of this electromagnet is the side from which the field lines emerge. Note then that the north pole of the external field is adjacent to the north pole of the disk and the south pole of the external field is adjacent tot eh south pole of the disk. The two magnets hence repel. This means that a force must be applied and work must be done to push the disk into the magnetic field. The energy that is put into the system through the mechanical work of pushing the disk is lost to heat from the eddy currents circulating in the disk.
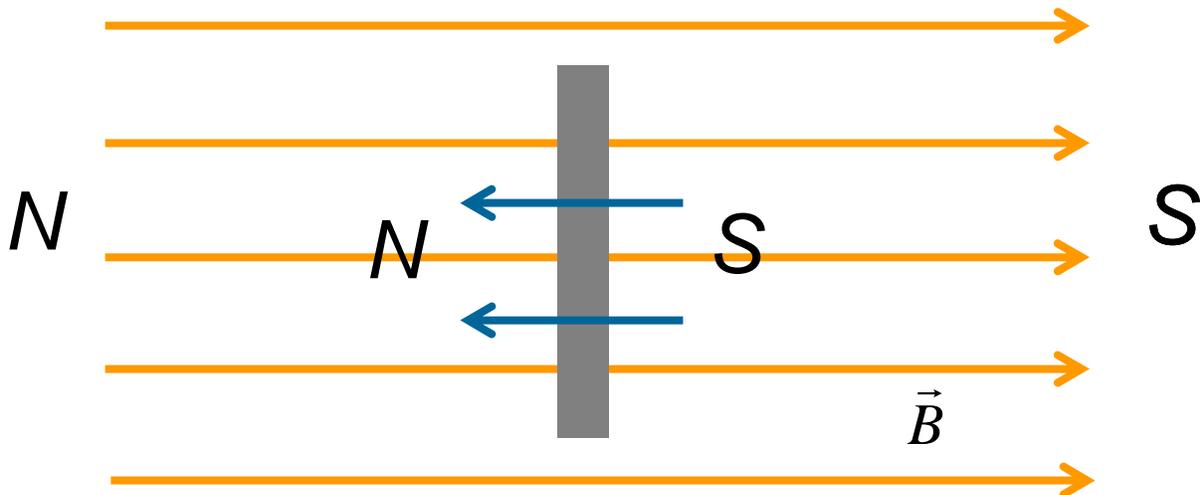
Figure 11.12. The induced magnetic field of a disk entering the field.

If the entire disk is within the external field, the magnetic flux through the disk is constant and no eddy currents are produced. However, if the disk leaves a region of magnetic field, there is an induced field in the disk oriented in the same direction as the external field.
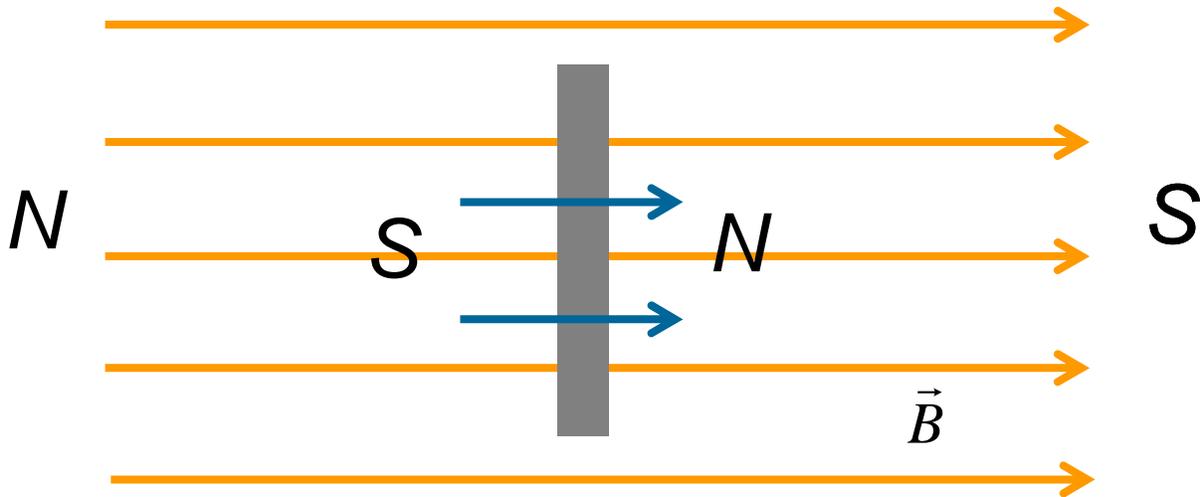
Figure 11.12. The induced magnetic field of a disk leaving the field.

In this case, the north and south poles are opposite each other and an attractive force opposes the motion of the disk out of the field.

In some applications, such as in transformers, eddy currents can cause a loss of energy that is undesirable. (We will discuss transformers later.) Iron cores of transformers are usually made of laminated sheets of iron that are insulated from each other in order to break up eddy currents. In other applications, eddy currents can be beneficial. For example, some scientific balances use eddy currents to damp unwanted motion.

Things to remember:
• When the magnetic field through a conductor changes, eddy currents are induced in the conductor.
• Eddy currents produce magnetic fields that oppose the motion of a conductor: if the conductor is moving into a stronger magnetic field, the force is repulsive. If the conductor is moving into a weaker magnetic field, the force is attractive.

11.8. Electric Motors.

We have already studied the basic physics of electric motors in Lesson 9 where we learned how a magnetic dipole in a magnetic field experiences a torque. The trick to making a motor is to keep the dipole rotating in the same direction all the time. There are many different types of motors; however, they are all basically the same in terms of how they work. The motors we will consider will all consist of a loop of wires, the dipole, rotating in the field of a permanent magnet, as illustrated in Fig. 11.13. The loop of wires is attached to a shaft about which the coil can rotate. The shaft and wires are collectively called the "armature" of the motor.
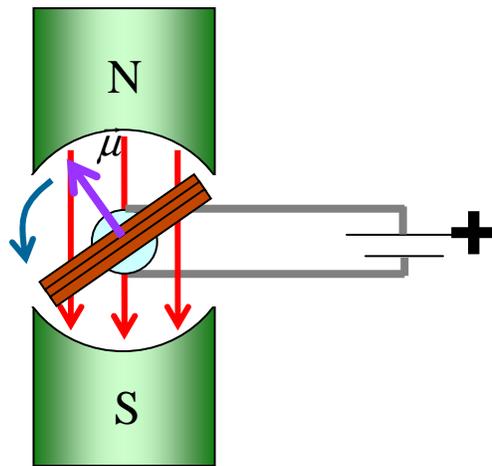
Figure 11.13. A simple motor.

As current flows through the loop, the loop becomes a magnetic dipole. The dipole experiences a torque that rotates the dipole moment into the direction of the external magnetic field. Once the dipole moment is parallel with the external magnetic field, the rotational inertia of the armature carries the loop onward, as shown in Fig. 11.14(a). The result is that the armature just oscillates
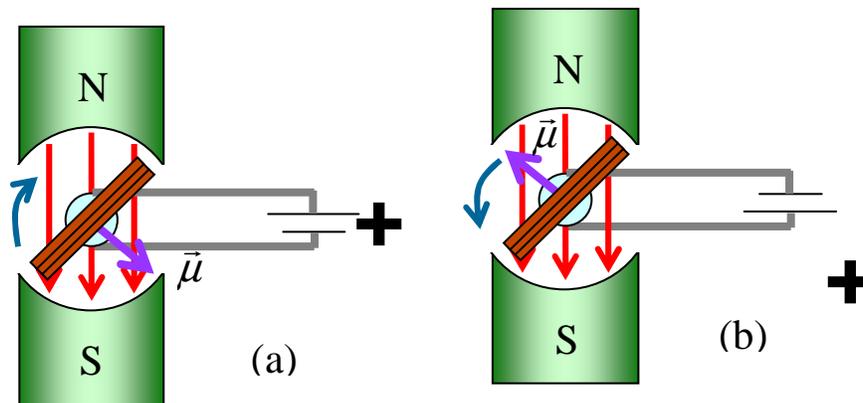

Figure 11.14. The need to reverse the current through the coil of the motor.

back and forth. In order to make a motor, we need to do something that keeps the armature turning in the same direction. One thing we could do is reverse the direction of the battery, as shown in Fig. 11.14(b). This reverses the direction of the current which, in turn, reverses the direction of the magnetic dipole moment of the loop. Hence, the torque keeps the armature rotating in a counterclockwise direction.

Before we consider what we need to do to repeatedly change the current direction in the coil, let's first consider how it is that we make an electrical connection between the coil and the battery to begin with. If we simply take the ends of the coil and attach them to a battery on the outside of the motor, the rotation of the motor would quickly twist and break the wires. To make a good electrical contact between the wires on the armature and the wires in the outside world,

we employ commutators and brushes, as shown in Fig. 11.15. A commutator is a conducting ring that passes around the shaft and is solidly connected to the shaft. As the shaft turns, the commutator turns as well. Wires from the loop pass along the shaft and are connected to the commutator. A brush is just a device that keepsthe non-rotating wire (from a battery, for example) in good electrical contact with the commutator. A typical brush consists of a block of graphite pressed against the commutator with a spring. As the graphite wears down, it also lubricates the commutator. More simple brushes can be made with something as simple as a spring steel clip that glides over the commutator.
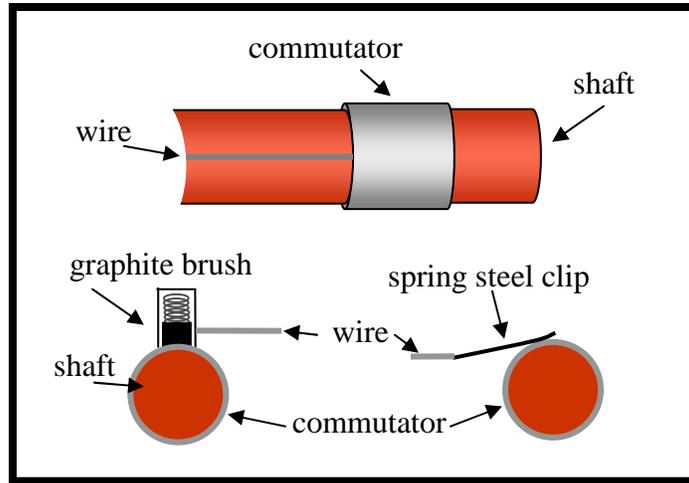

Figure11.15. Commutators and brushes.

If we wish to connect a power supply to a loop, we need to connect two wires to the aramture's coil. We can do that with a "double commutator" as shown in Fig. 11.16.
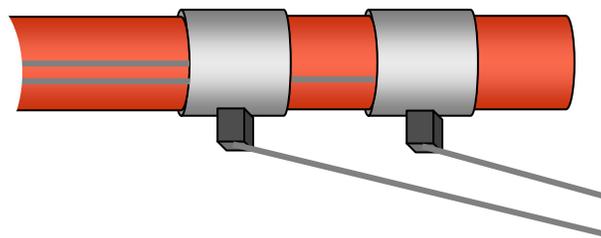

Figure 11.16. A double commutator.

Now, let's return to the question of changing the sign of the current flowing through the loop of wire. Clearly, we need something more efficient than turning a battery around every half cycle. The easiest answer to the problem is to use a power supply that automatically changes the direction of the current at a regular rate. An alternating current (AC) power supply fits this requirement, as it provides a sinusoidally varying voltage. All we need to do is connect an AC power supply to the brushes of a motor with a double commutator, and the motor will continue to

turn. Note, however, that the angular speed of the motor is fixed by the frequency of the power supply.

Another way around the problem allows us to use a direct current (DC) power supply such as a battery. This method uses what is known as a "split commutator." In this arrangement, a simple commutator is cut in half and fixed onto the shaft so that each hald is insulated from the other. The positive terminal of the battery is connected to one brush and the negative terminal to the other brush. In Fig. 11.17 we see how a split commutator works. In the first drawing, the positive terminal of the battery is connected to commutator A. After the armature has rotated a half turn, commutator A is connected to the negative terminal of the battery.
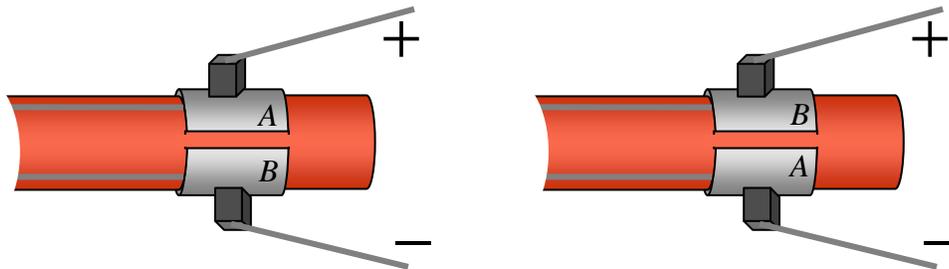

Figure 11.17. Split commutators at two different times.

Things to remember:
• An electric motor can be made by placing a current loop in an external magnetic field.
• The direction of the current through the loop must reversed every half cycle to keep the armature of the motor rotating in the same direction.
• AC motors use a double commutator with an AC power supply to reverse the current direction.
• DC motors use a split commutator with a DC power supply to change the current direction.

11.9. Electric Generators.

Electric generators are just electric motors that run in reverse. We provide some kind of mechanical force (falling water, a steam turbine, a hand-crank, etc.) to turn a coil of wire in a magnetic field. The rotating coil then produces an electric field by induction.

Running an AC motor in reverse produces an AC generator (or alternator). The EMF produced by the generator is given in Example 11.3 above.

$$\varepsilon = NBA\,\omega \sin \omega t$$

where
ε is tne EMF in volts (V).
N is the number of loops in the coil.
B is the external magnetic field strength in tesla (T).
A is the area of each loop in m^2.
ω is the angular velocity of the armature in *rad/sec*.

This tells us that the output of an AC generator is a simple sine function, as shown in Fig. 11.18.
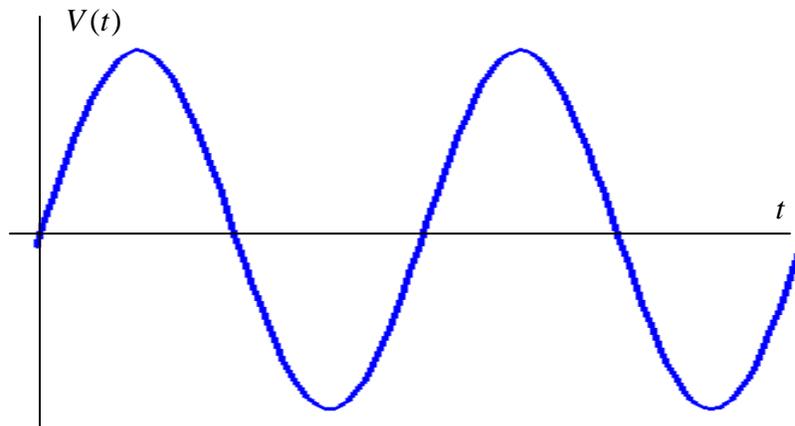

Figure 11.18. The sine wave output of an AC generator.

If we use a split commutator on our generator, then the direction of the current reverses every half cycle. The result is $V(t) = NBA\omega|\sin \omega t|$.
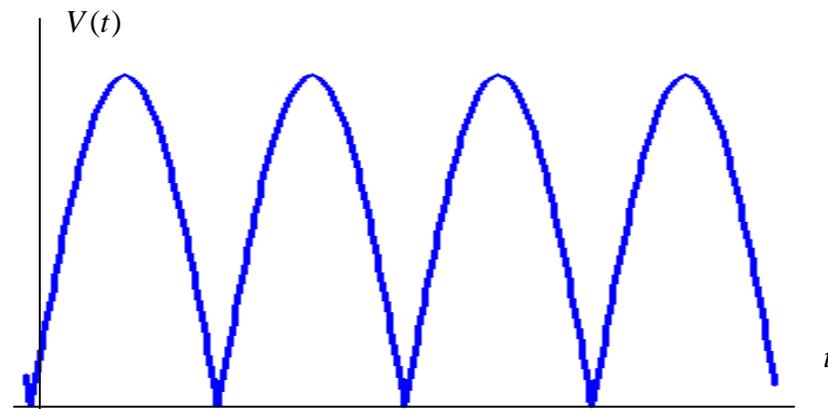

Figure 11.19. The output of a generator with split commutators.

While this isn't exactly a DC voltage, it is an EMF that always pushes charges around a circuit in the same direction.

We could make a simple modification to the basic motor/generator we first introduced by adding a second coil of wires on the armature, as shown in Fig. 11.20. By adding the current output of these two coils together, we get a voltage that somewhat resembles a CDC output.
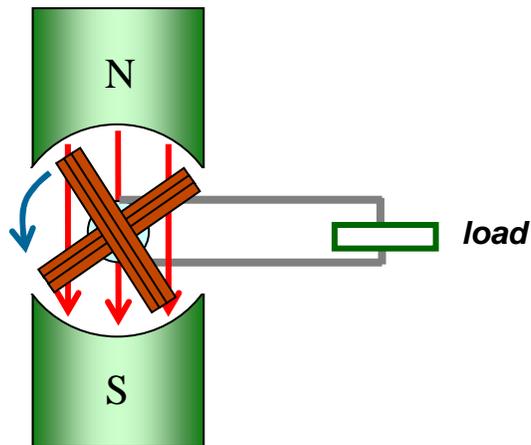
Figure 11.20. A "DC" generator with two coils.

The output of this generator is illustrated in Fig. 11.21. The red lines are the outputs of the individual coils and the blue line is the sum of the two.
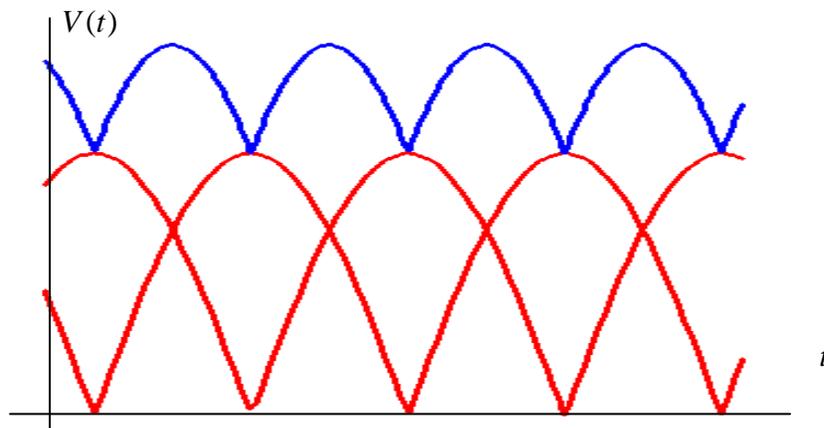

Figure 11.20. The output of the "DC" generator of Figure 11.20.

Things to remember:
• Generators are essentially like motors except that we turn the armature by some mechanical means to induce an EMF in the coil.
• A generator with double commutator produces an AC voltage.
• A generator with multiple coils and split commutators for each coil can produce a quasi-DC voltage.