

## Unit 2 Review

### 5.1

- Electric field lines go radially outward from positive charges and radially inward toward negative charge.
- The field is proportional to the number of field lines per unit area (taken on a section of a perpendicular surface).
- The field lines of a point charge are uniformly distributed at all angles.
- Field lines are a geometrical way of expressing the physics of Coulomb's Law.
- We usually use "flat" drawings of field lines, but we need to remember that this is just a cartoon version of the real, three-dimensional world.

### 5.2

- For point charges, perpendicular surfaces are spheres with the charge at the center.
- Perpendicular surfaces are also equipotential surfaces for any static distribution of charge.
- Adjacent surfaces in a field contour for static electric charges are separated by a fixed voltage.

### 5.3

- The magnetic field of a long, current-carrying wire is  $B(r) = \mu_0 i / 2\pi r$ .
- The direction of magnetic field lines around a wire is given by the right-hand rule.
- Magnetic field lines are similar to electric field lines in that they are close together where the field is stronger and the field is tangent to the field lines.
- The magnetic field lines have to be drawn at correct locations, much as electric field contours.

### 5.4

- The  $I/r$  dependence of the magnetic field of a wire is naturally built into the magnetic field contour of the wire.
- The direction of a perpendicular surface is defined as the direction of the magnetic field on the surface.

### 5.5

Electric Field	Magnetic Field
Point source	Current source
Field lines begin and end on charges	Field lines form closed loops
Field lines extend to infinity	Field lines remain localized
$q/r^2$ dependence in field lines	$i/r$ dependence in field contours
Perpendicular surfaces are closed; that is, the surfaces enclose the charge	Perpendicular surfaces are open
Perpendicular surfaces remain localized	Perpendicular surfaces extend to infinity
Perpendicular surfaces are equipotential surfaces	Perpendicular surfaces have no physical interpretation

## 5.6

- The near field and far field are similar to the fields of point charges.
- The direction of a field line at any point in space is the direction of the force on a small, positive test charge.

## 5.7

- Be sure that you understand each of the drawings in this section and that you can make correct drawings of electric fields for two or three point charges.
- Correct drawings must have the correct near field behavior, the correct far field behavior.
- Correct drawings must have field lines starting on positive charges and ending on negative charges.
- Correct drawings must have the number of field lines be proportional to the charge.

## 5.8

- In this course, there are three important symmetries we need to consider: radial spherical, radial cylindrical, and planar.
- By invoking symmetry, we can simplify many problems.

## 5.9

- As with charges, it is best to consider first the near field and the far field.
- If two parallel wires carry current in the same direction, the intermediate field typically has peanut-shaped field lines.
- The intermediate field lines will be typically closer to a wire on one side than the other. Look at the whether the fields of the individual wires are in the same direction or in opposite directions to determine which side has the larger field.

## 5.10

- In flat diagrams the electric field lines resemble the magnetic field contours and the magnetic field lines resemble the electric field contours.

## 5.11

1. Since electric field lines and magnetic field contours carry the geometry of the field laws, we concentrate on drawing these. Remember that, although the drawings look similar for both cases, they represent much different things in three dimensions.

2. The field lines and contours in the near field and the far field are those of point charges or current-carrying wires.

3. In between these extremes remember:

Field lines (contours) never cross.

Field lines (contours) are closer together where the field is stronger.

Field lines (contours) originate and end on charges (wires).

4. Draw electric field contours and magnetic field lines perpendicular to the electric field lines and magnetic field contours. Try to space them so that they are closer together where the field is stronger.

5. Practice drawing field lines and contours for a number of simple arrangements of charges and currents.

6.1

- Capacitors have charge  $+Q$  on one conductor and charge  $-Q$  on a second conductor.
- Capacitance is defined by the relation  $Q = CV$ .
- In steady state, the voltage across a capacitor is equal in magnitude to the applied voltage.
- In steady state, no current flows in the branch of a circuit containing a capacitor.

6.2

- The electric field inside a capacitor is uniform so  $E = V/d$ .
- The capacitance of a parallel-plate capacitor is  $C = \frac{\epsilon_0 A}{d}$ .
- Memorize the capacitance expression, and you can deduce  $V$  and  $E$  from it.

6.3

- The energy it takes to charge a capacitor becomes available as the capacitor is discharged.
- The energy stored in a capacitor is  $U = \frac{1}{2} CV^2$ .
- We can think a capacitor's energy as being stored in the electric field. The energy density (energy per unit volume) of an electric field is  $u = \frac{1}{2} \epsilon_0 E^2$ .

6.4

- A dielectric increases the capacitance by a factor  $\kappa$ .  $\kappa \geq 1$ .
- If the charge on a capacitor is fixed, a dielectric reduces the voltage by a factor  $\kappa$ . It does this by reducing the electric field in the capacitor.
- If the voltage on a capacitor is fixed, a dielectric increases the charge on a capacitor by a factor  $\kappa$ . It does this by pulling charge onto the plates from the battery.

6.5

- The equations for adding capacitors in series and parallel are the reverse of those for resistors.
- Capacitors in parallel have the same voltage.
- Capacitors in series have the same charge.

6.6

- Capacitors charge and discharge with a time constant  $\tau = RC$ .
- The time constant is the time it takes a decaying exponential function to fall to  $1/e$  of the function's initial value if decreasing or to rise to  $1 - 1/e$  of its final value if increasing.
- In circuits where capacitors are charged and discharged, the charges and currents are of one of two forms:

$$f(t) = f(0)e^{-t/\tau}$$

$$f(t) = f(\infty)(1 - e^{-t/\tau})$$

### 7.1

- A Gaussian surface is a closed surface – a surface that encloses a volume.
- Gauss's Law of Electricity: The net number of electric field lines passing through a Gaussian surface is proportional to the total charge enclosed by the Gaussian surface.
- Gauss's Law is equivalent to Coulomb's Law because it just an observation based on field lines, and field lines are geometrical representations of Coulomb's Law.

### 7.2

- Charge densities may be volume charge densities ( $\rho = \text{charge/volume}$ ), surface charge density ( $\sigma = \text{charge/area}$ ), or linear charge density ( $\lambda = \text{charge/unit length}$ ).

### 7.3

- We can use Gauss's Law with symmetry to calculate the electric field of spherically symmetric charge distributions. Know how to do that!
- Outside the charge distribution, the field is the same as the field of a point charge with the same total charge.
- Inside a hollow sphere with a spherically symmetric charge density, the electric field is zero.
- At a radius  $r$  inside the distribution, the field is the same as the field of a point charge that has the same charge as the charge *inside* a Gaussian surface of radius  $r$ .

### 7.4

- In a static situation, all net charge on a conductor must lie on its outer surface.
- Know how to prove that this is true using Gauss's Law in conjunction with the fact that the static electric field inside a conductor is zero.

### 7.5

- Magnetic field lines always form closed loops.
- Gauss's Law of Magnetism: the net number of magnetic field lines passing through a Gaussian surface is always zero.

### 7.6

- You can use Coulomb's Law to find the electric field at radius  $r$  for any spherically symmetric charge distribution as long as you remember that the charge that contributes to the field is only the charge on the inside of a sphere of radius  $r$ .
- In general, you can find the enclosed charge by integrating over the charge density:
 
$$q = \int \rho dV.$$

### 7.7

- By appropriate slicing of areas and volumes, you reduce many integrals to a simpler form.
- You must integrate, however, over all variables that appear in the integrand.
- If you slice a disk into annular rings, the area of each ring is  $dA = 2\pi r dr$ .

### 7.8

- The volume of a cylindrical shell is  $2\pi r\ell dr$ .
- The volume of a spherical shell is  $4\pi r^2 dr$ .
- Memorize these expressions, as we will make frequent use of them!

### 7.9

- The mathematical function that describes the number of field lines is called the flux.
- You should remember that flux is essentially field  $\times$  area.
- In general, we need to account for the angle of the surface with respect to the field and non-uniformities in the field when we calculate the flux.
- Know the formal definition of flux  $\Phi = \int \vec{E} \cdot d\vec{A}$

### 7.10

- Gauss's law is always true.
- Gauss's Law in integral form is  $\Phi_E = \oint \vec{E} \cdot d\vec{A} = \frac{q_{enc}}{\epsilon_0}$ . This is just a mathematical way of saying that the net number of electric field lines passing through a Gaussian surface is proportional to the charge enclosed in the surface

### 7.11

- In a few cases of high symmetry, Gauss's Law reduces to  $EA = \frac{q_{enc}}{\epsilon_0}$ .
- These cases are limited to charge distributions with spherical, cylindrical, or planar symmetry.
- In these cases, we can use Gauss's Law to find electric fields. The only tricky part is integrating over the charge distribution to find the enclosed charge.
- This is a very important section. Know it well!

### 8.1

- Amperian loops are closed loops.
- Ampère's Law: The net number of surfaces pierced by an Amperian loop is proportional to the current passing through the loop.
- We always go around Amperian loops in a counterclockwise direction. We also define positive current to be "out" and negative current to be "in."
- Ampère's Law is equivalent to the relation  $B = \mu_0 i / (2\pi r)$ .

### 8.2

- Current density  $j$  is the current per unit area passing through a wire. Current density may vary from region to region in a wire; however, in typical wires, current density is quite uniform.
- We can integrate current density to get total current.  $I = \int j dA$

### 8.3

- The magnetic field is zero inside a hollow wire with radially symmetric current density.
- The magnetic field  $B(r)$  inside a solid wire with radially symmetric current density is the magnetic field of the core (the part of the wire with radius  $< r$ ) alone.

### 8.4

- The line integral is a quantity that is proportional to the number of surfaces (belonging to a field contour) pierced by a line segment.
- When we choose a magnetic field line for the line segment and the magnetic field is constant on the field line, the line integral reduces to  $\Lambda = B\ell$ .
- The general form for the line integral is  $\Lambda = \oint \vec{B} \cdot d\vec{\ell}$ .

### 8.5

- Gauss's Law in integral form is  $\Lambda_B = \oint \vec{B} \cdot d\vec{\ell} = \mu_0 \int j dA = \mu_0 i_{enc}$ .
- In practical applications this reduces to  $B\ell = \mu_0 \int j dA$

### 8.6

- Be able to use Ampère's Law to find the magnetic field inside and outside a wire with radially symmetric current density.
- Be able to use Ampère's law to find the magnetic field in a solenoid and in a torus.
- You do not need to memorize the formulas for these magnetic fields.

### 8.7

- Be able to find integral expressions for the electric field and the electric potential of straight rods and charged, circular loops on the axis.
- Be able to find integral expressions for the magnetic fields of straight wire segments and circular wire loops on the axis.
- You do not need to know how to evaluate the integrals that arise.

### 8.8

- The definition of divergence:  $div \vec{E} = \lim_{\Delta v \rightarrow 0} \frac{\Phi_E}{\Delta v}$
- The definition of curl:  $[curl \vec{B}(\vec{r})]_x = \lim_{\Delta a \rightarrow 0} \frac{\Lambda_{B,x}}{\Delta a}$
- Gauss's Law of Electricity:  $\nabla \cdot \vec{E} = \frac{\rho}{\epsilon_0}$
- Gauss's Law of Magnetism:  $\nabla \cdot \vec{B} = 0$
- Ampère's Law:  $\nabla \times \vec{B} = \mu_0 \vec{j}$
- Gradient operator:  $\nabla = \hat{x} \frac{\partial}{\partial x} + \hat{y} \frac{\partial}{\partial y} + \hat{z} \frac{\partial}{\partial z}$
- Divergence operator:  $div \vec{E} = \nabla \cdot \vec{E} = \frac{\partial E_x}{\partial x} + \frac{\partial E_y}{\partial y} + \frac{\partial E_z}{\partial z}$
- Curl operator:  $curl \vec{B} = \nabla \times \vec{B}$